ML Based Newton preconditioner

Antoine Lechevallier

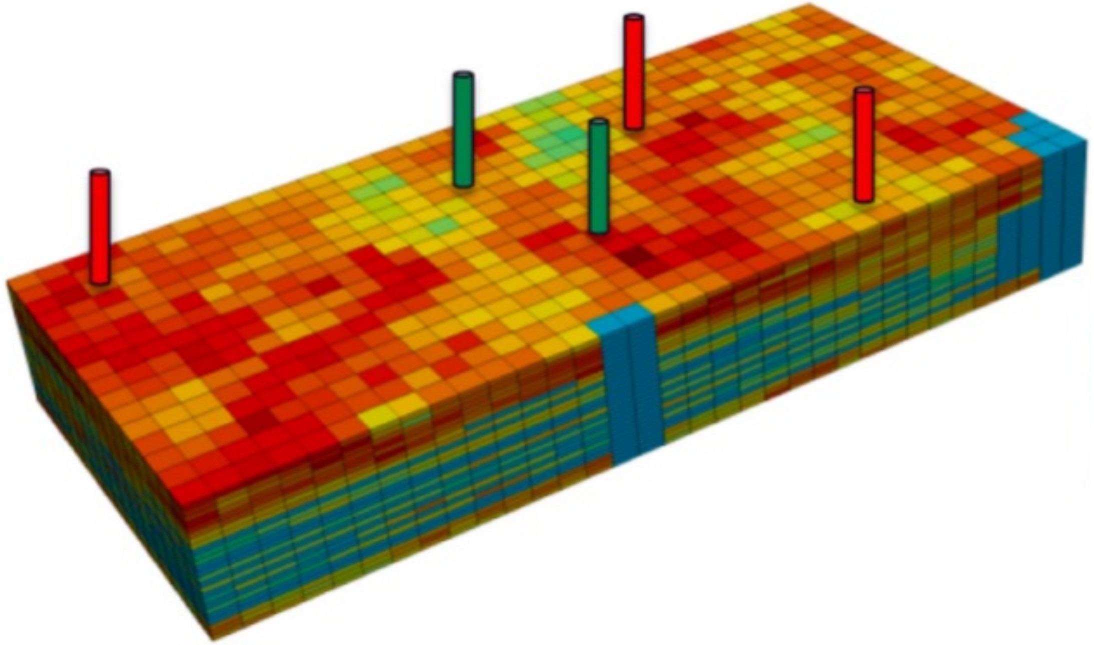
Supervised by:

Frédéric Nataf[1]
Thibault Faney[2]
Sylvain Desroziers[3]
Eric Flauraud[2]

[1] Laboratoire Jacques-Louis Lions
[2] IFP Energies nouvelles
[3] Michelin

Numerical simulation of Carbon storage

Advantages	Drawbacks
Safety	Model uncertainties
Parametric studies	Parameters uncertainties
Cost and time efficiency	Computational resources



2) Bicheng Yan et al.

CO_2 storage requires important amount of simulations

→ Modify parameters to test scenarios

Explore recent breakthroughs in Artificial Intelligence to accelerate numerical simulation

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Outline

- Mathematical model and numerical resolution
- Global Hybrid Newton
- Results
- Conclusion

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Mathematical model and Numerical resolution

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Mathematical model

- Incompressible two-phase flow in porous medium

$$\phi \frac{\partial}{\partial t}(1 - S) + div(v_w) = 0, \quad \phi \frac{\partial}{\partial t}(S) + div(v_g) = q_g, $$

Mass conservation

$$v_w = -\frac{Kkr_w(1 - S)}{\mu_w}\nabla P, \quad v_g = -\frac{Kkr_g(S)}{\mu_g}\nabla P,$$

Darcy's law

$$S = S_g = 1 - S_w,$$

Closure law

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Numerical Resolution

1

P, **S** as main unknowns

$S_w = 1 - S$ auxiliary unknown

2

Discretization

- Finite volume in space
- Implicit Euler in time

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Numerical Resolution

1 **P**, **S** as main unknowns

$S_w = 1 - S$ auxiliary unknown

2 Discretization

- Finite volume in space
- Implicit Euler in time

- Fully Implicit scheme

$$\begin{cases} R_g(S^{n+1}, P^{n+1}) & = 0, \\ R_w(1 - S^{n+1}, P^{n+1}) & = 0. \end{cases}$$

➔ **Non-linear** system of equations solved using **Newton's method**

➔ **Unconditionnaly stable** but large time steps can prevent Newton's method from converging

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Numerical Resolution

1 **P**, **S** as main unknowns

$S_w = 1 - S$ auxiliary unknown

2 Discretization

- Finite volume in space
- Implicit Euler in time

Python library:

Yads: Yet Another Darcy Solver

- Fully Implicit scheme

$$\begin{cases} R_g(S^{n+1}, P^{n+1}) & = 0, \\ R_w(1 - S^{n+1}, P^{n+1}) & = 0. \end{cases}$$

➔ **Non-linear** system of equations solved using **Newton's method**

➔ **Unconditionnaly stable** but large time steps can prevent Newton's method from converging

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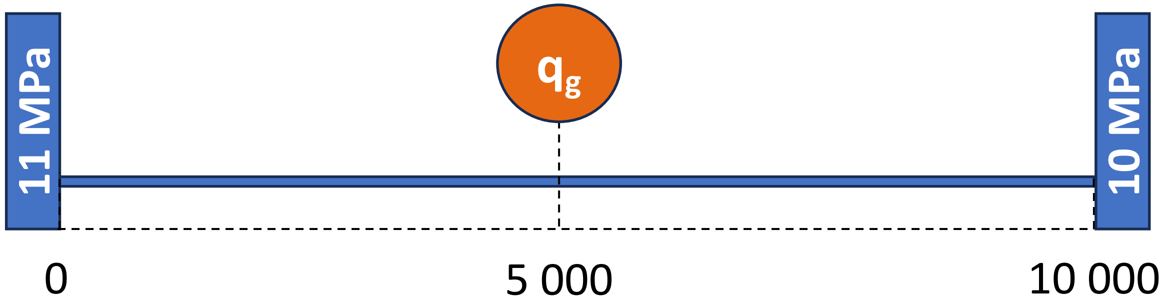
Reservoir geometries

1D

200 cells

Variable parameters :

- Well injection flow rate: $\mathbf{q_g}$
- Time-step: **dt**



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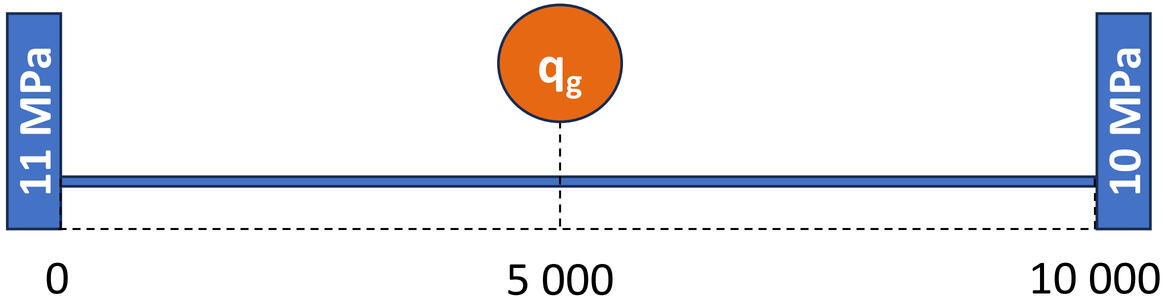


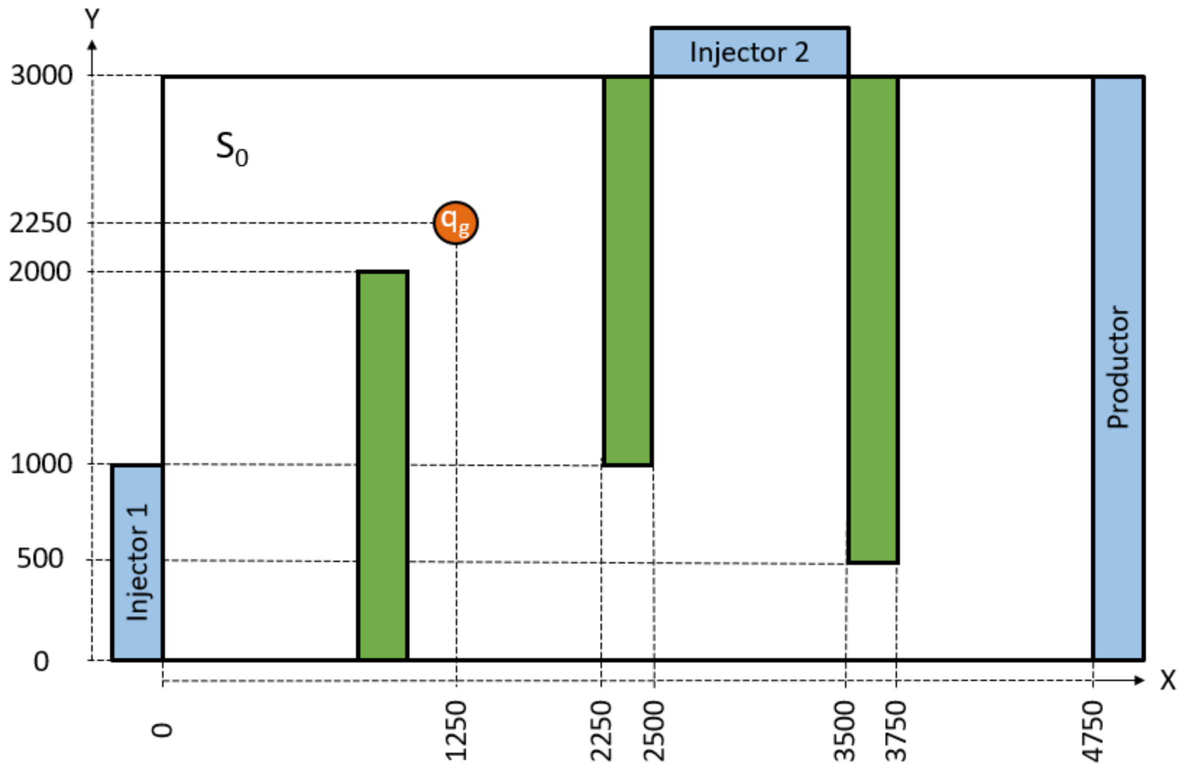
2D

SHPCO2 benchmark[1]

95 X 60 cells

Variable parameters :

- Initial saturation: $\mathbf{S_0}$
- Well injection flow rate: $\mathbf{q_g}$
- Time-step: **dt**



1) *Florian Haeberlein 2011 Time Space Domain Decomposition Methods for Reactive Transport — Application to CO2 Geological Storage.*

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Global Hybrid Newton

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Global Hybrid Newton

- Scenario :
  1. Well opening
  2. Well closing

  Transport

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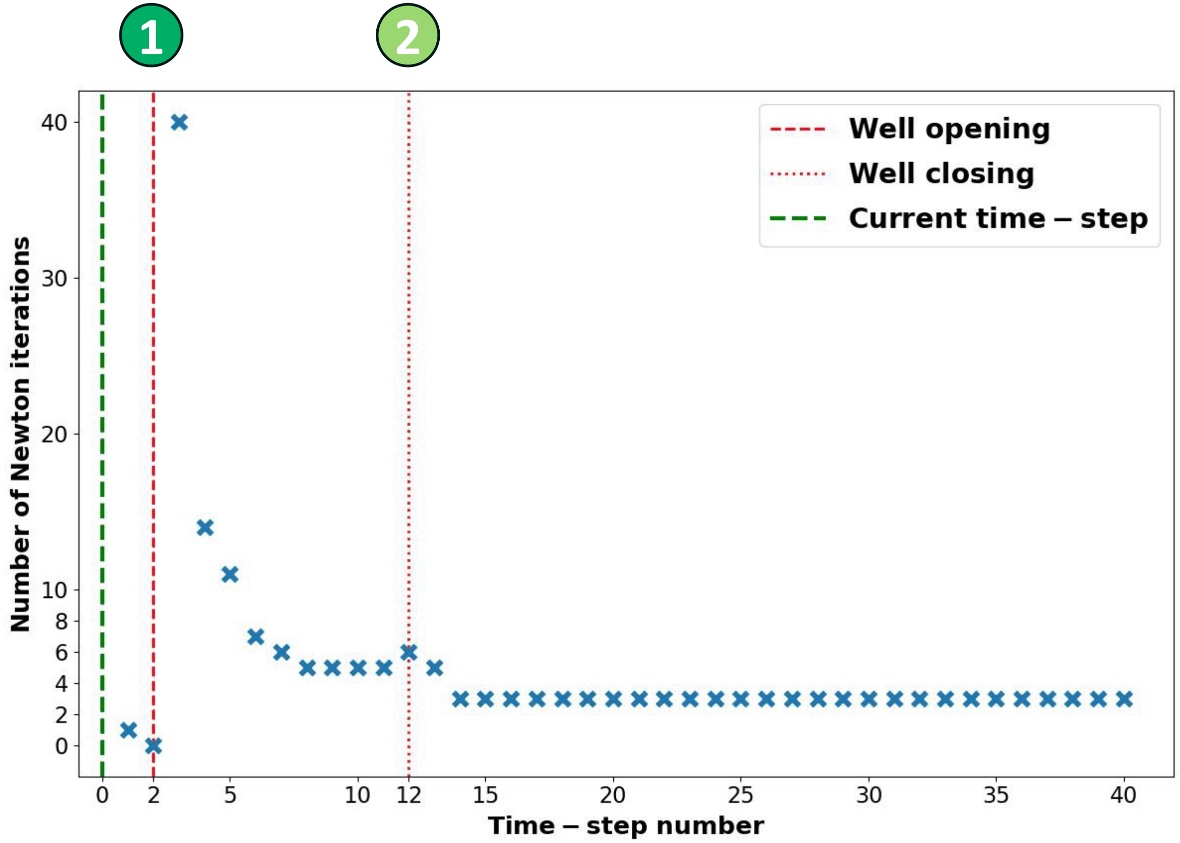
Global Hybrid Newton

- Scenario :
  - 1 Well opening
  - 2 Well closing
  - Transport

Computational cost?



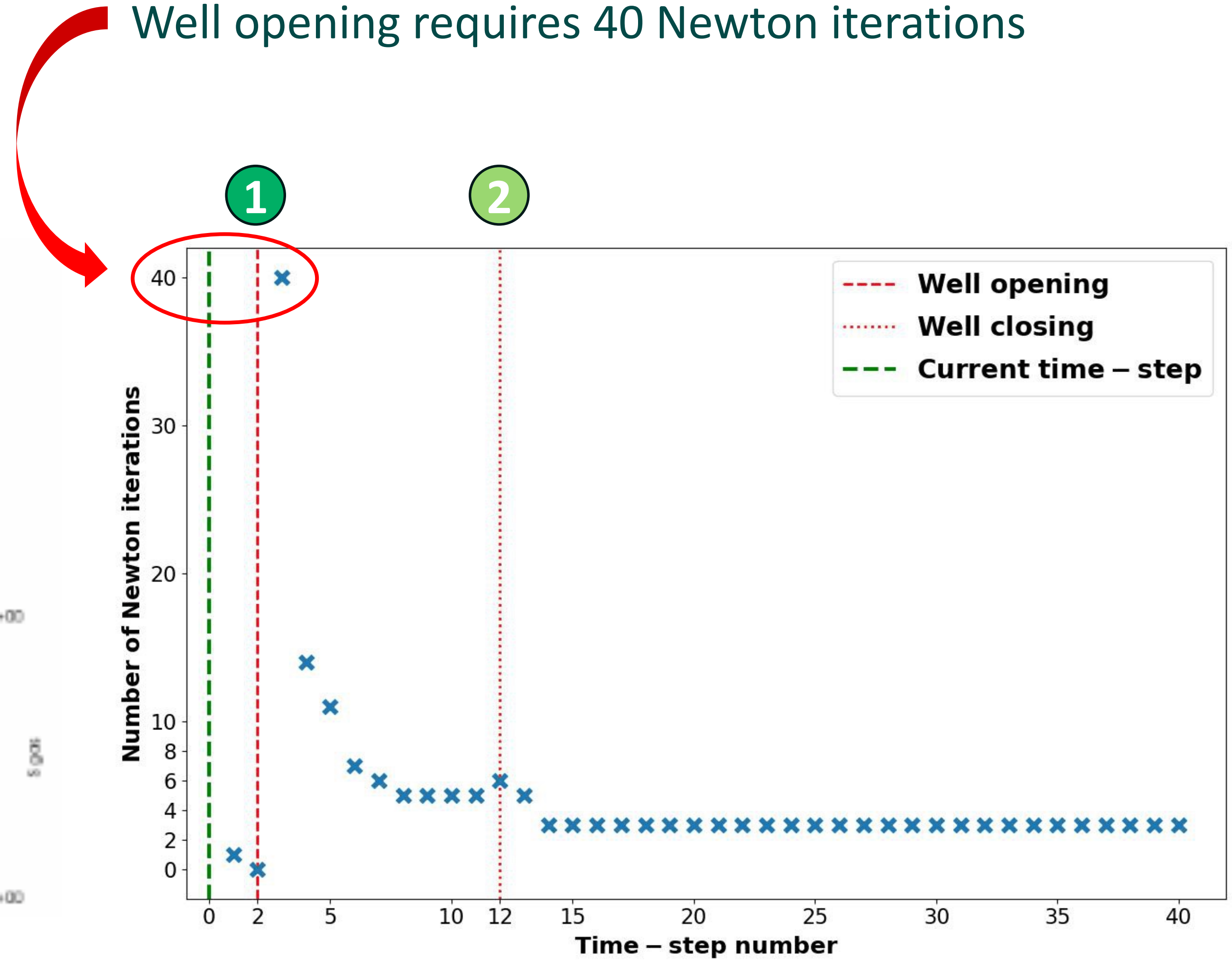
Global Hybrid Newton

- Scenario :
  1. Well opening
  2. Well closing

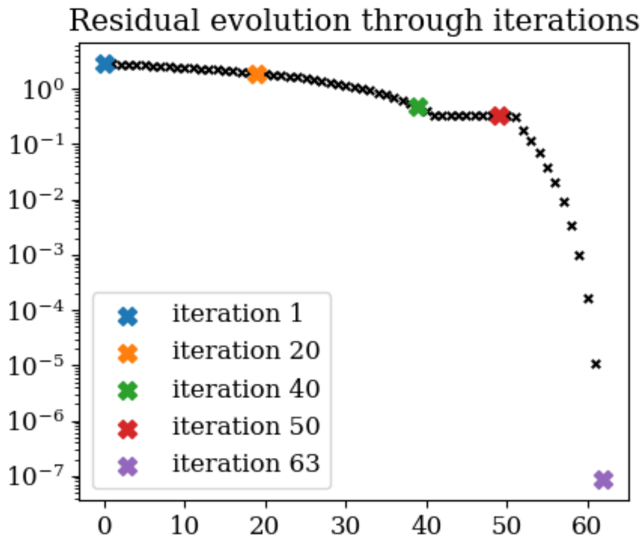
  Transport

Computational cost?

Well opening requires 40 Newton iterations



Hybrid initialization: 1D example



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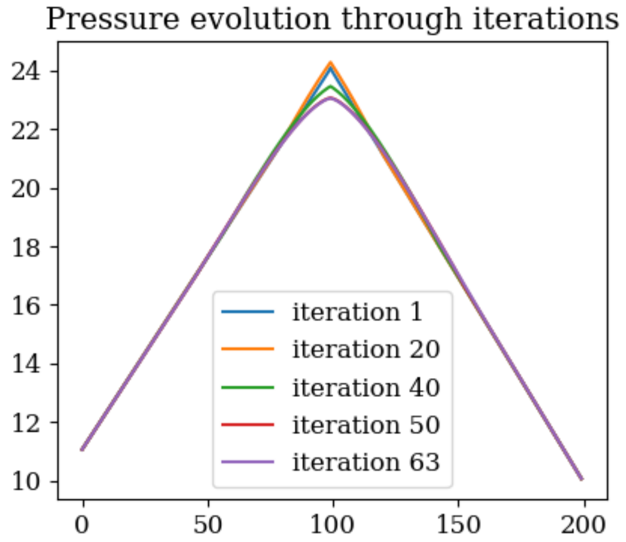
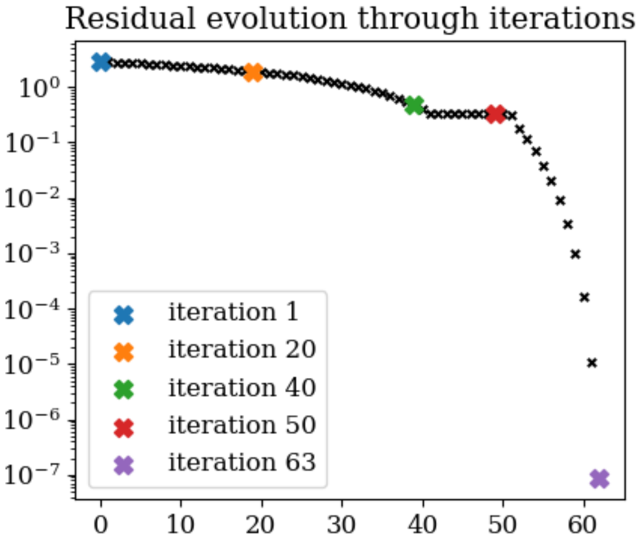
Hybrid initialization: 1D example





Pressure | **Implicit Pressure Solver**

→ Solve linear system

$$div(v_w(P^{n+1}, 1 - S^n)) = 0, \quad div(v_g(P^{n+1}, S^n)) = q_g.$$

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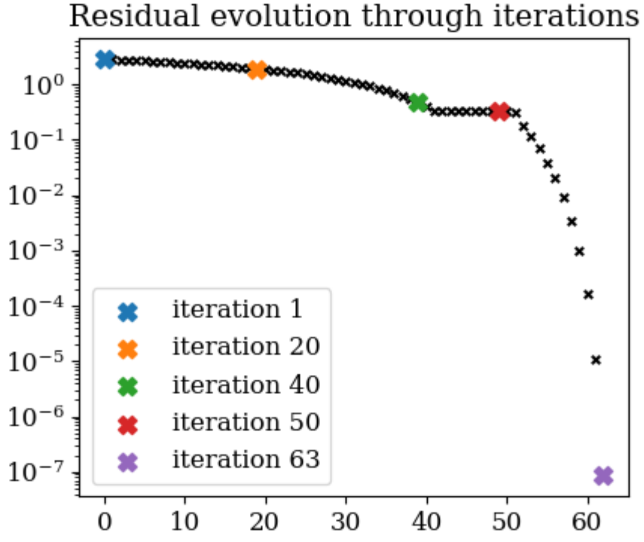
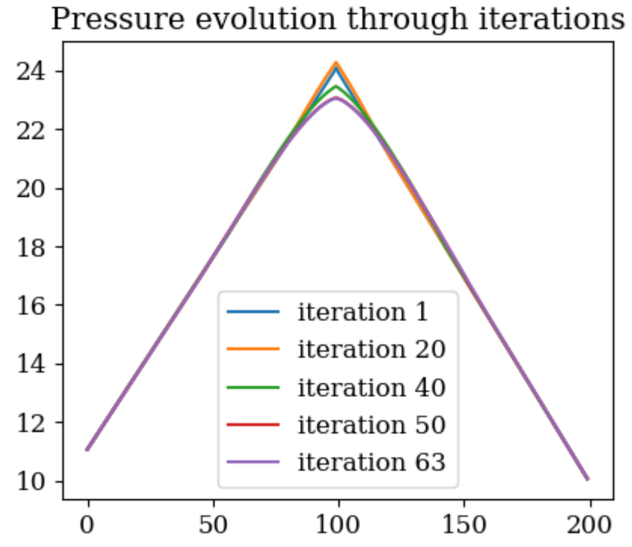
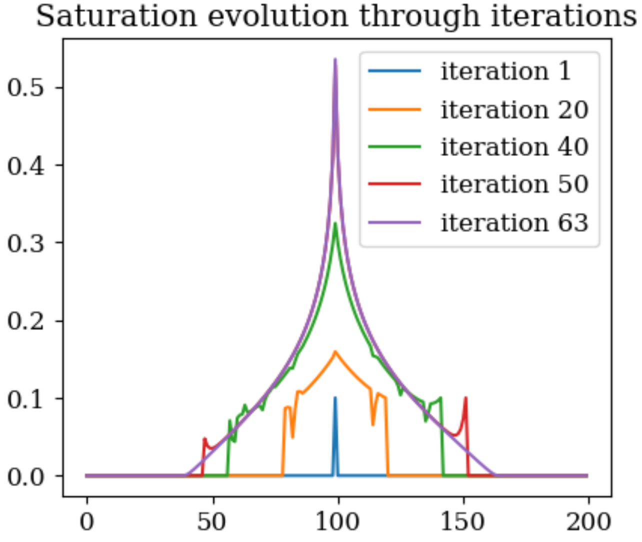
Hybrid initialization: 1D example







Pressure | **Implicit Pressure Solver**

→ Solve linear system

$$div(v_w(P^{n+1}, 1 - S^n)) = 0, \quad div(v_g(P^{n+1}, S^n)) = q_g.$$

Saturation | Well events are similar in space and time

→ **Machine Learning model**

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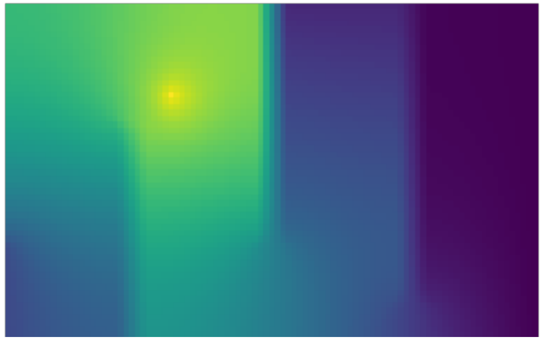
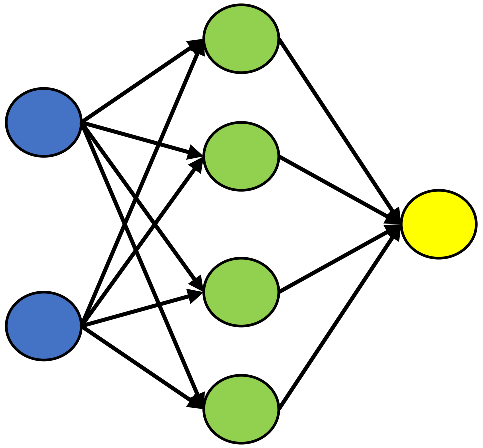
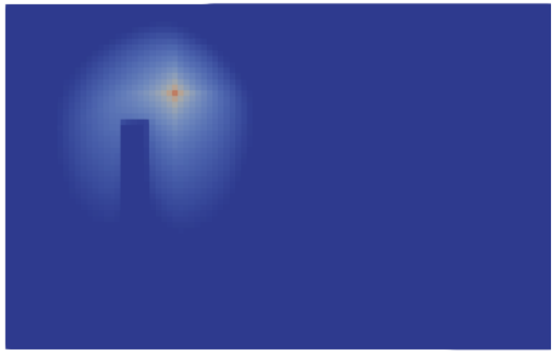
Workflow

Generate data → **Supervised training of Saturation model** → **Plug saturation model in Newton's method**







→ Cover large range of well events

→ Fourier Neural operator

$$S_{guess} = f_{\theta}(P_{imp}, S, q_g, dt)$$

→ Compare Hybrid with Standard Newton's method

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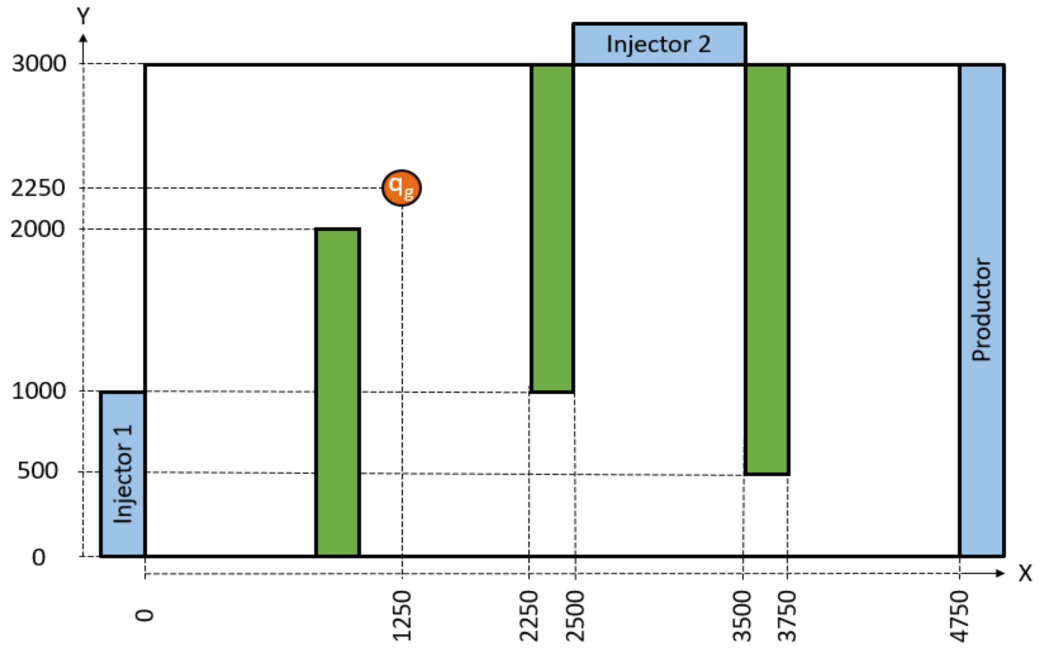
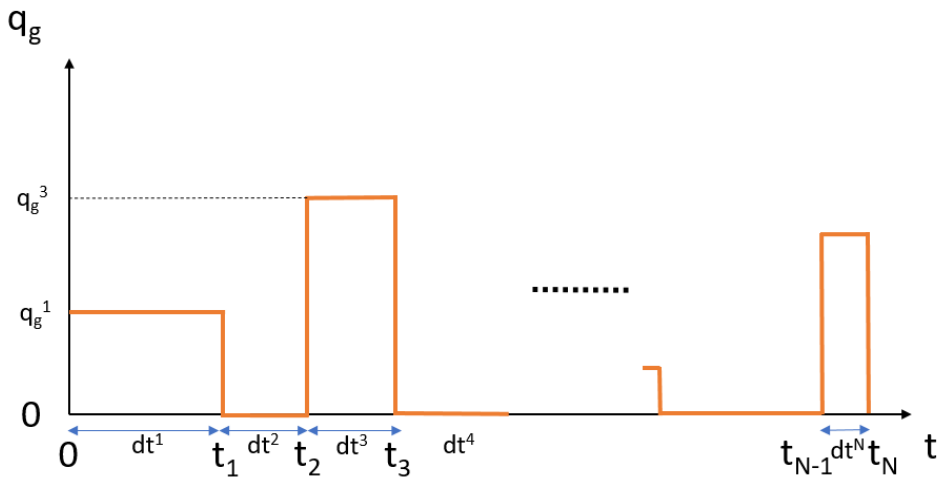
Data Generation

- **Variable parameters**
  - Well injection flow rate: $\mathbf{q_g} \in [10^{-5}, 10^{-3}]$ m²/s
  - Well opening time: **dt** ∈ [1, 10] years
  - Reservoir gas saturation: **S** ∈ [0, 0.6]

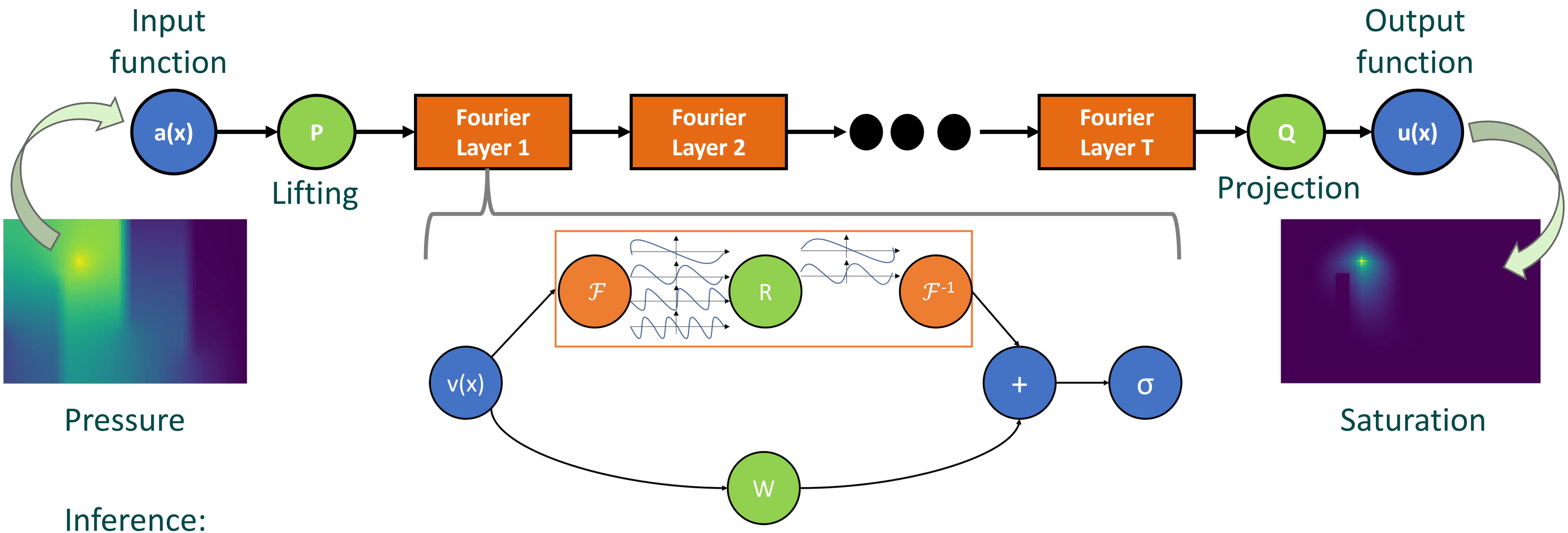
- **Generate realistic saturations S**
  - Consecutive well opening and closing
  - 3 600 parameters combination
  - 18 000 samples

- **Computational cost**
  - Run in parallel using MPI
  - 360 CPUs
  - ≈36 hours





Fourier Neural Operator[1]



Inference:

Majority of the inference cost is in the Fourier Transform !

Fast Fourier Transform → Nlog(N)

1) Zongyi Li, et al 2020. Fourier neural operator for parametric partial differential equations.

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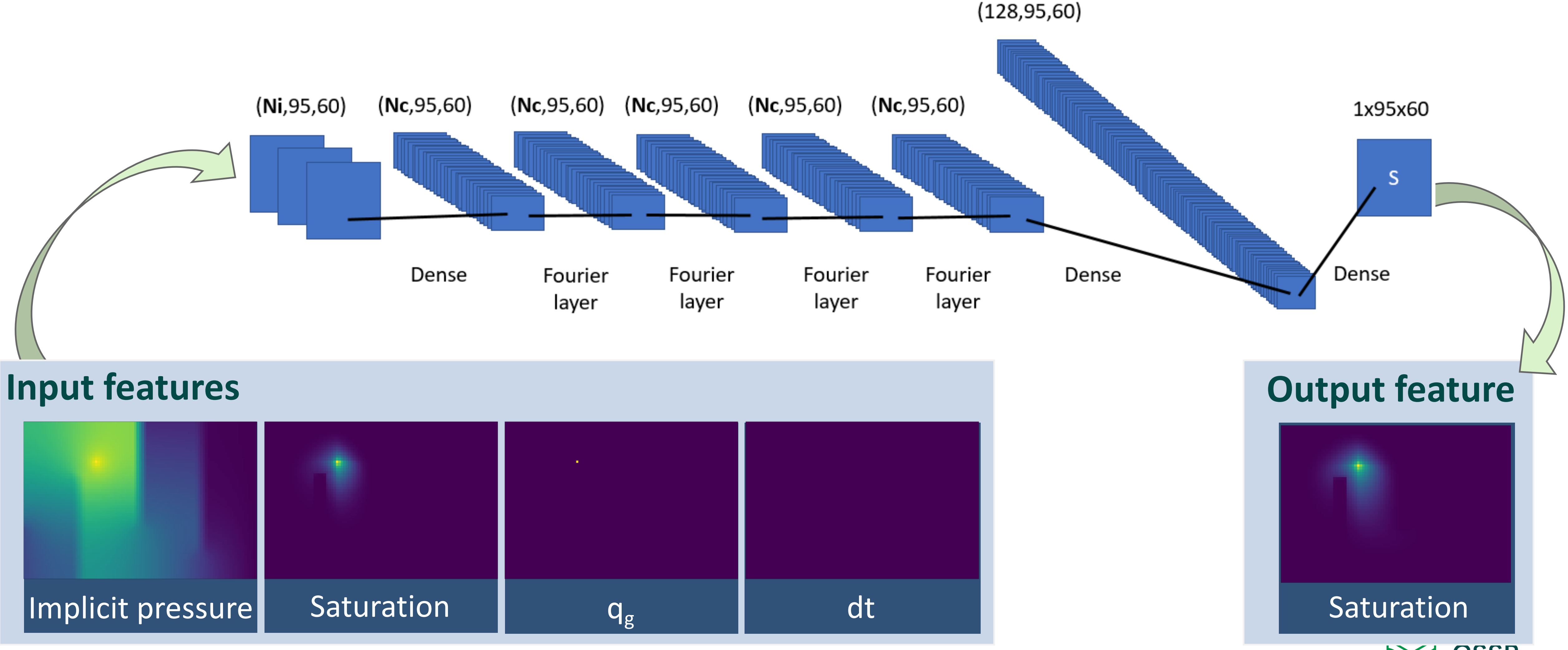
Saturation model - Architecture



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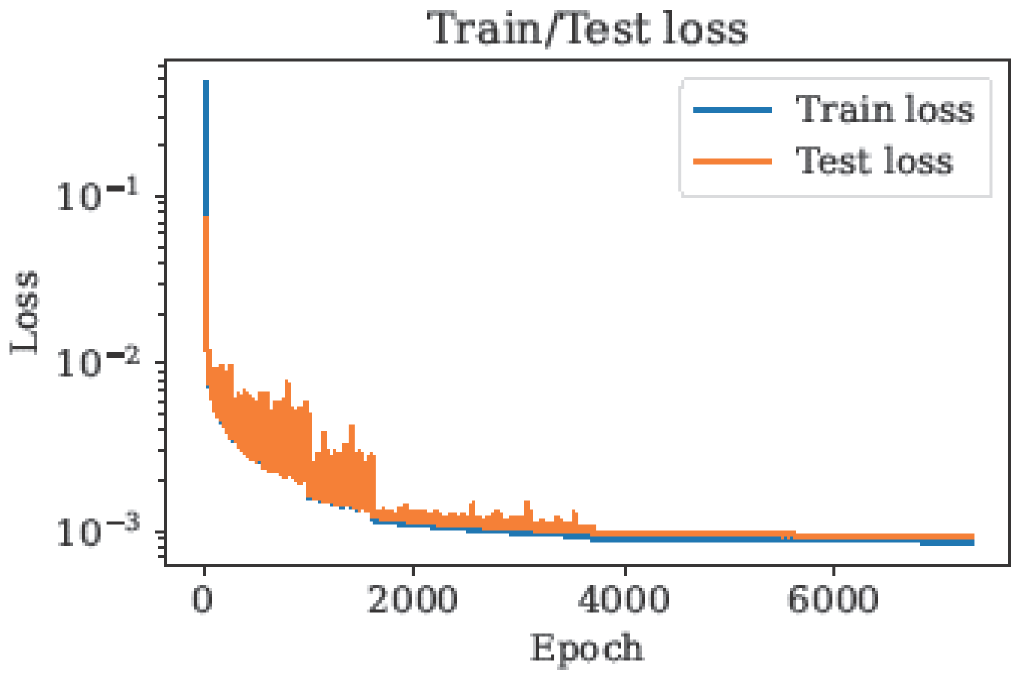
Saturation model - Training

- Dataset
  - 80% train / 20% test
- Computational cost
  - 132 hours on NVIDIA V100 GPU
- Hyperparameters
  - Loss: $\frac{1}{N}\sum_{i=1}^{N} \frac{\|s_i - \hat{s}_i\|_2}{\|s_i\|_2}$
  - Batch size: 128
  - Adam optimizer
  - Starting learning rate: 10^{-4}
  - Momentum: 0.9
  - Weight decay: 10^{-4}



Best model	epoch 7295
Test loss	**8.7×10^{-4}**
Train loss	**9.3×10^{-4}**

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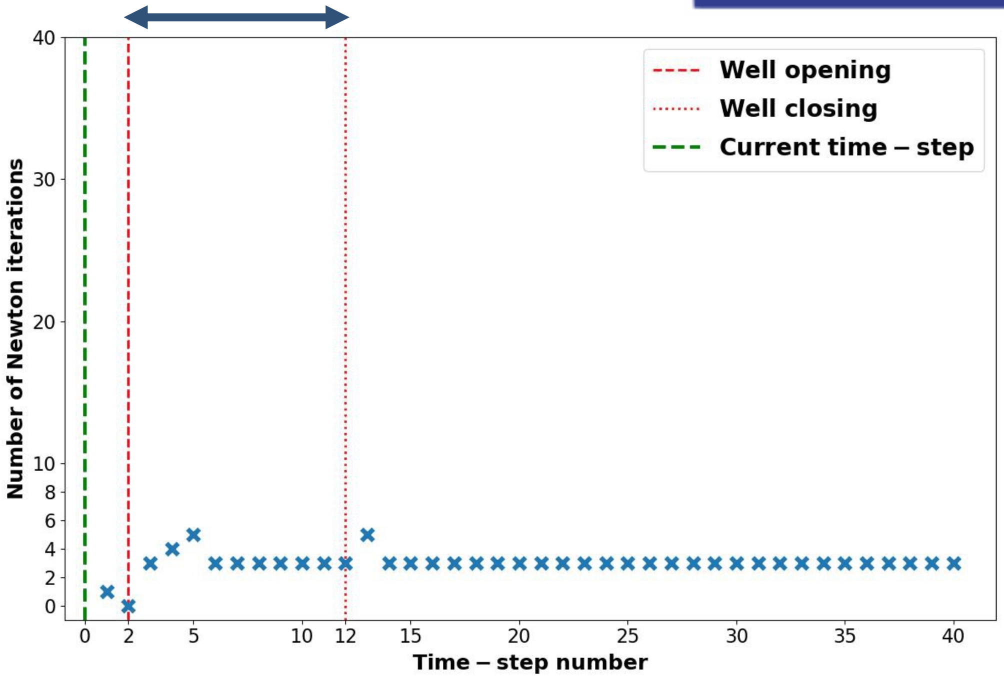
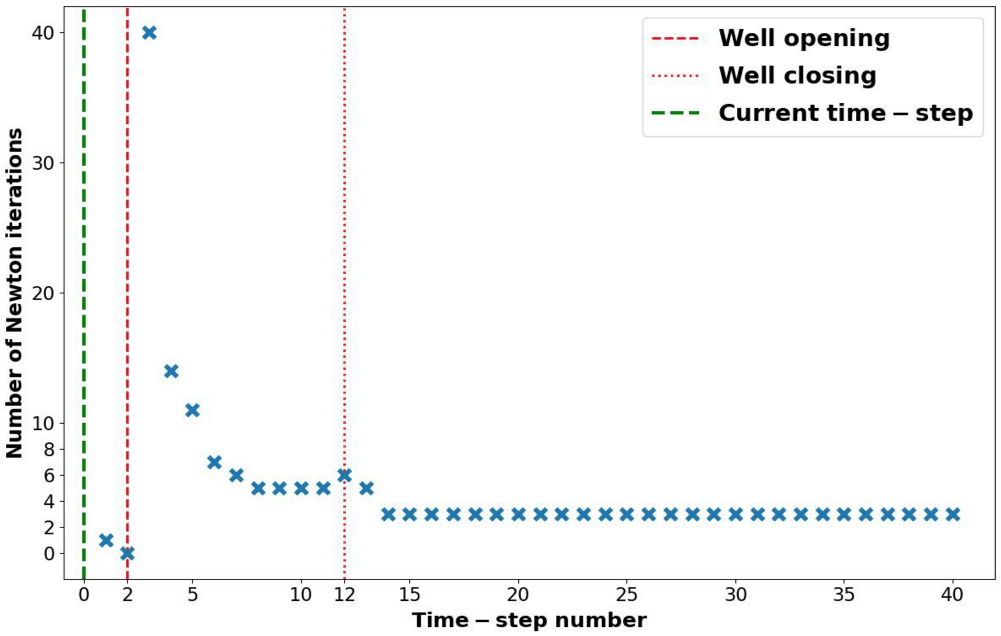
Results

Hybrid Newton



Standard Newton

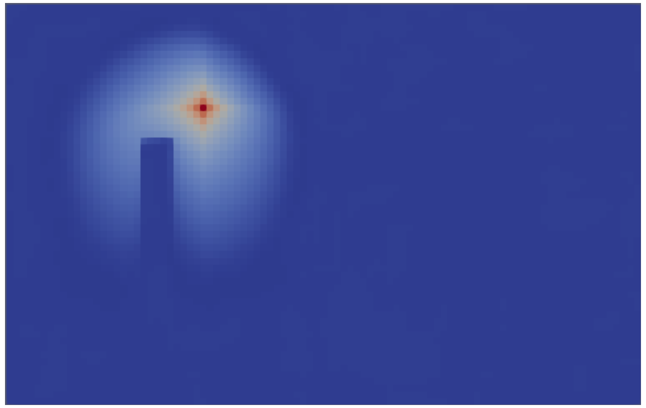


R

Results

Saturation initializations :

Standard



Hybrid

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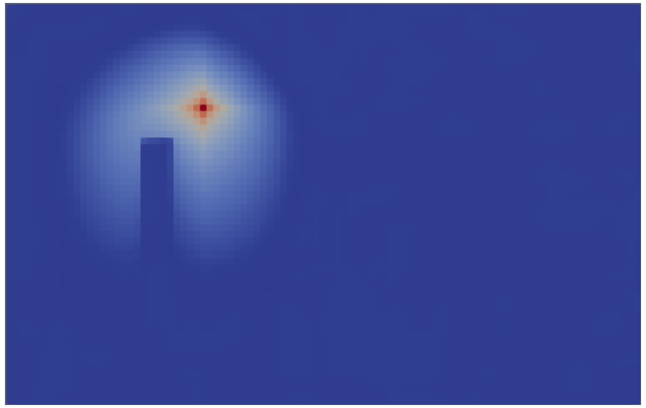
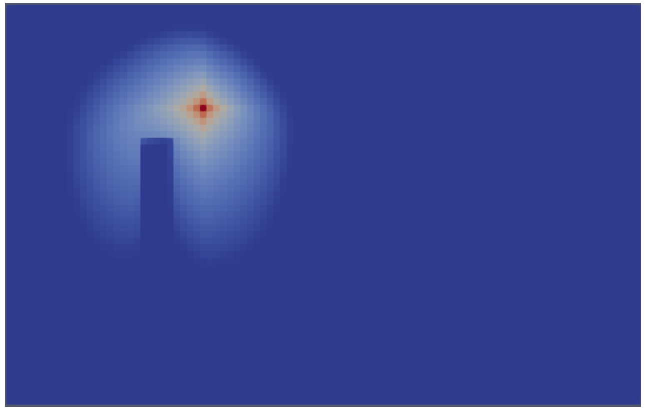
Results

Saturation initializations :

Standard



Hybrid



Solution

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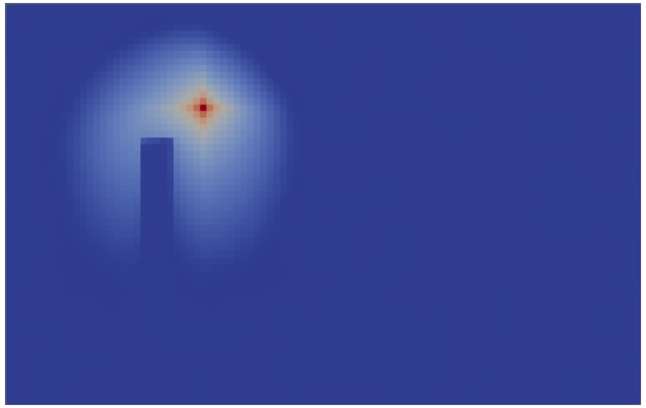
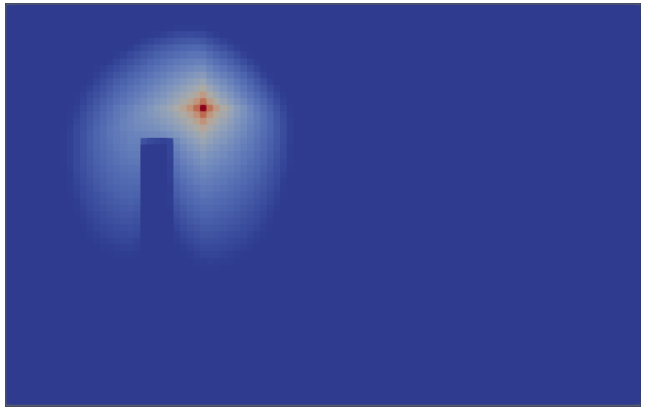
Results

Saturation initializations :

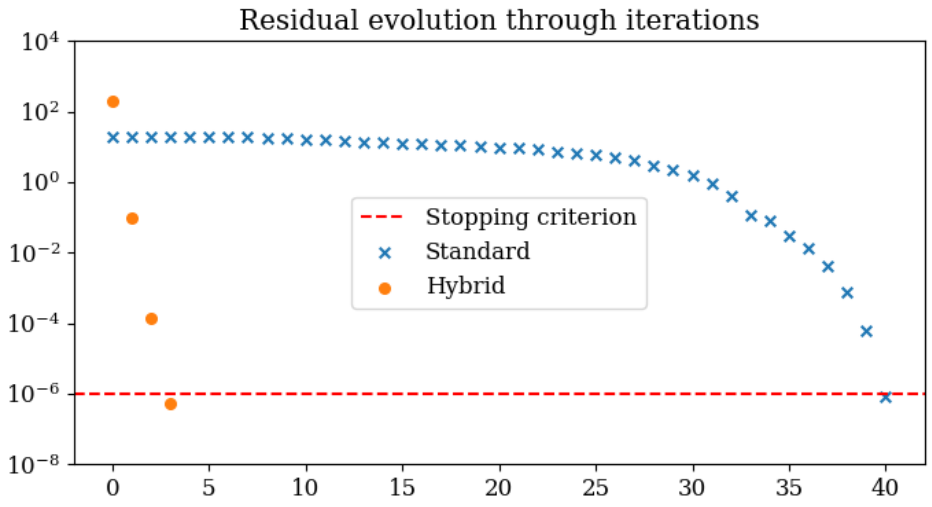
Standard



Hybrid



Solution



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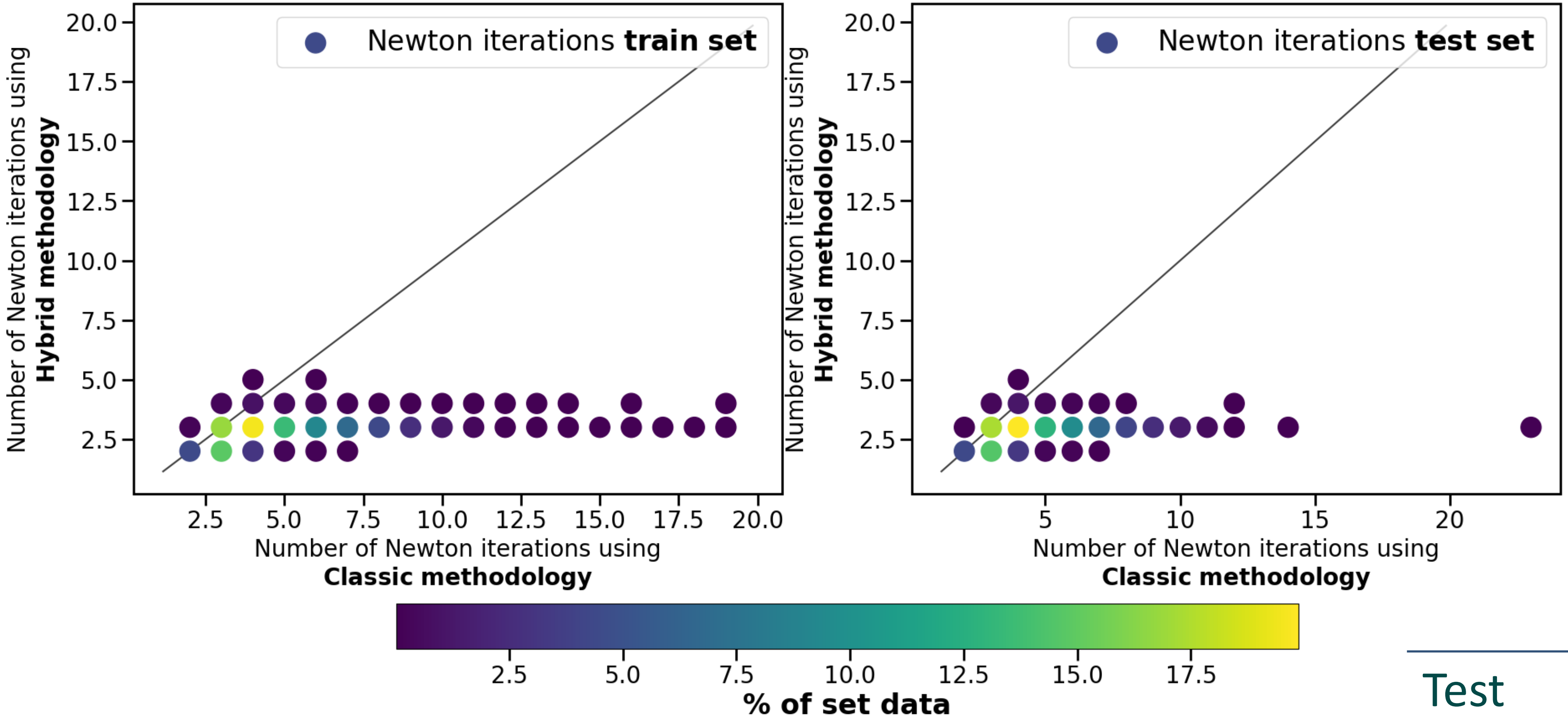
Results



	Acceleration
Test	**39%**
Train	**39%**

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Summary of Global Hybrid Newton

Advantages	Drawbacks
Acceleration of numerical simulation for a large range of well events	Offline phase expensive : • Data generation • Model training
Online phase affordable	Constant well location

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Conclusion and perspectives

- Integration of Hybrid Newton preconditioning in OPM
- Challenges:
  - More complex physical model
  - **Training data generation cost**
  - **Multi-well handling**
  - Heterogeneities
  - Discretization
- Current developments:
  - Local Hybrid Newton → Reduce training cost / Multi-well handling
  - Generalized well model → Reduce Dataset generation cost

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1. Hybrid Newton
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2. Other citations
    - *Bicheng Yan, Bailian Chen, Dylan Robert Harp, Wei Jia, Rajesh J. Pawar, A robust deep learning workflow to predict multiphase flow behavior during geological CO2 sequestration injection and Post-Injection periods, Journal of Hydrology, Volume 607, 2022, 127542, ISSN 0022-1694, https://doi.org/10.1016/j.jhydrol.2022.127542.*
3. *Github repository*
    - *https://github.com/PINN-Well-opening-and-closing-events/Yads.git*

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Research for optimal reservoir operations and value generation in the green transition

Energy-efficient petroleum production

Strategies to synchronize water injection with intermittent energy

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Scalable solutions for subsurface storage of hydrogen, air, and heat

CCS: Carbon storage

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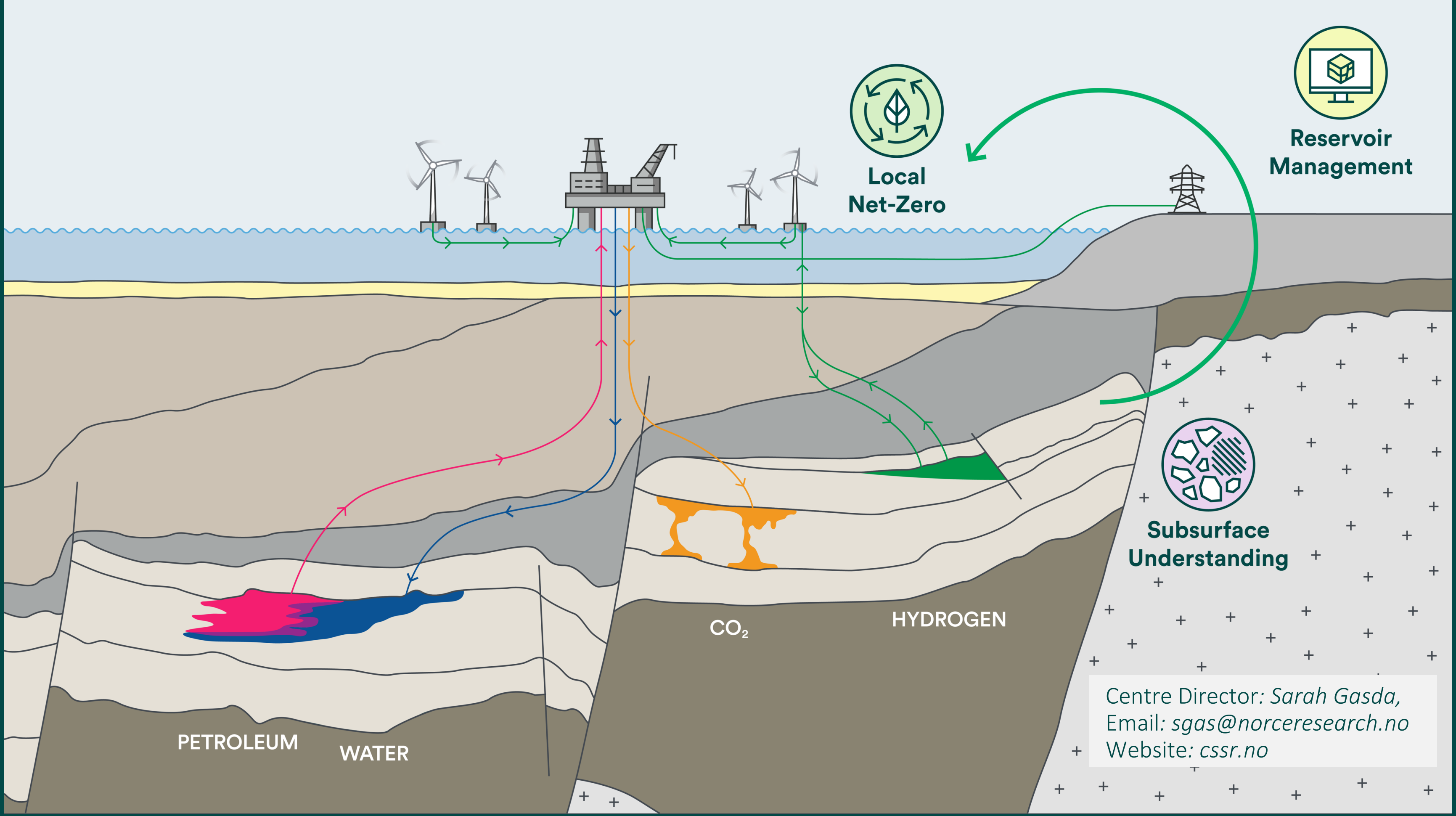
Interdisciplinary research and education

Reservoir physics, geosciences and applied mathematics



Centre Director: *Sarah Gasda,*
Email: *sgas@norceresearch.no*
Website: *cssr.no*

Forskningsrådet

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