

Parallelization of Multisegment Wells: Enhancing simulation flexibility

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What are multisegment wells?





Simulation flexibility



- Partitions are critical for the performance!
- The new default partitioner handles well info better \rightarrow performance improvement!
- Before: Partitions had to keep multisegment wells on one process.
- With distributed multisegment wells: Partitions can be independent of the wells.
- Enable with --allow-distributed-wells=true.
- Spoiler¹: The well-independent version of the new default partitioner slightly outperforms the well-dependent one!

¹for the cases I have tested

Steps to parallelize multisegment wells



- A Linear system solved in each timestep in flow, Schur complement
- B Equations and resulting part of the linear system for standard and multisegment wells
- C Linear system for multisegment wells in parallel
- D Performance results
- E Outlook



• reservoir + well equations define a large system of nonlinear equations

$$R(x) = \vec{O}$$

 $dim(x) = dim(R(x)) = \underbrace{\#\text{grid cells} \cdot \#\text{primary variables}^{1}}_{\#\text{reservoir unknowns}} + \underbrace{\#\text{wells} \cdot \#\text{unknowns per well}}_{\#\text{well unknowns}}$

• solve $R(x) = \vec{O}$ using a Newton-Raphson type method, i.e., in each iteration: calculate Jacobian $J(\cdot) \in \mathbb{R}^{dim(x) \times dim(x)}$ of R and solve

$$J(x_n)(x_{n+1}-x_n)=-R(x_n)$$

¹one primary variable for each phase, so 3 primary variables for a three-phase black oil system



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• solve $R(x) = \vec{O}$ using a Newton-Raphson type method, i.e., in each iteration: calculate Jacobian $J(\cdot) \in \mathbb{R}^{dim(x) \times dim(x)}$ of R and solve

$$J(x_n)x_{n+1} = J(x_n)x_n - R(x_n)$$



reservoir + well equations define a large system of nonlinear equations

$$R(x) = \vec{O}$$

 $dim(x) = dim(R(x)) = \underbrace{\#\text{grid cells} \cdot \#\text{primary variables}^{1}}_{\#\text{reservoir unknowns}} + \underbrace{\#\text{wells} \cdot \#\text{unknowns per well}}_{\#\text{well unknowns}}$

 solve R(x) = 0 using a Newton-Raphson type method, i.e., in each iteration: calculate Jacobian J (·) ∈ ℝ^{dim(x)×dim(x)} of R and solve

 $J(x_n)x_{n+1}=r_n$



• reservoir + well equations define a large system of nonlinear equations

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• solve $R(x) = \vec{O}$ using a Newton-Raphson type method, i.e., in each iteration: calculate Jacobian $J(\cdot) \in \mathbb{R}^{dim(x) \times dim(x)}$ of R and solve

$$\begin{pmatrix} \mathsf{A} & \mathsf{C} \\ \mathsf{B} & \mathsf{D} \end{pmatrix} \begin{pmatrix} \mathsf{x}_{reservoir} \\ \mathsf{x}_{wells} \end{pmatrix} = \begin{pmatrix} \mathsf{r}_{reservoir} \\ \mathsf{r}_{wells} \end{pmatrix}$$





- A: Jacobian of reservoir equations w.r.t. reservoir unknowns
- B_j, C_j, D_j for each well, $j = 1, \ldots, \#$ wells
- *B_j*: Jacobian of equations for well *j* w.r.t. reservoir unknowns
- C_j: Jacobian of reservoir equations w.r.t. unknowns of well *j*
- *D_j*: Jacobian of equations for well *j* w.r.t. unknowns of well *j*



• Do not solve the whole system

$$\begin{pmatrix} A & C_1 & C_2 & \dots \\ B_1 & D_1 & & \\ B_2 & D_2 & \\ \vdots & & \ddots \end{pmatrix} \cdot \begin{pmatrix} x_r \\ x_{w_1} \\ x_{w_2} \\ \vdots \end{pmatrix} = \begin{pmatrix} r_r \\ r_{w_1} \\ r_{w_2} \\ \vdots \end{pmatrix}$$

as one, instead:

• For *x_r* solve

$$\left(A-\sum_{j=1}^{\#wells}C_jD_j^{-1}B_j\right)x_r=r_r-\sum_{j=1}^{\#wells}C_jD_j^{-1}r_{w_j}.$$

• For each x_{w_i} , solve the small system

$$x_{w_j} = D_j^{-1} \left(r_{w_j} - B_j x_r \right).$$

AC1C2xrxrB1D1xw1
$$xw2$$
 $rw1$ B2D2D2

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B: Well equations for a standard well



• one conservation equation per phase α :



- for a three-phase black oil system: $\alpha \in \{ \text{water, oil, gas} \} = \{ \text{w, o, g} \}$
- one control equation, e.g. for a prescribed bottom hole pressure:

$$p_{bhp} - p_{bhp}^{target} = 0$$

• unknowns for each well for a three-phase black oil system:



B: Well equations for a standard well



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• unknowns for each well for a three-phase black oil system:

$$\underbrace{Q_t(Q_w,Q_o,Q_g)}_{(d_w,d_w,d_w)}, \underbrace{F_w(Q_t,Q_w),F_g(Q_t,Q_g)}_{(d_w,d_w)} \quad \text{and} \quad p_{bhp}$$

weighted total flow rate weighted fractions of water and gas

 $\,
ightarrow \,$ four equations and four unknowns per standard well

B: Linear system for standard wells for 3 phases



- A is a #cells × #cells BCRS-matrix¹, inner matrices are 3 × 3
- *B_j* is a 1 × #*cells* BCRS-matrix, inner matrices are 4 × 3
- C_j is a #cells × 1 BCRS-matrix, inner matrices are 3 × 4
- *B_j* and *C_j* only contain entries for the cells that are perforated by the well *j*
- B_j and C_i^T have same sparsity pattern
- D_j is a 1 \times 1 BCRS-matrix, inner matrix is 4 \times 4



¹BCRS = Blocked Compressed Row Storage collection of small dense inner matrices

B: Well equations for a standard \rightarrow multisegment well \frown OPM

• one conservation equation per phase α (water, oil, gas) :



• one control equation for the top segment, e.g. for a prescribed bottom hole pressure:

$$p_{bhp} - p_{bhp}^{target} = 0$$

• unknowns for each standard well for a three-phase black oil system:

 $Q_t(Q_w, Q_o, Q_g), F_w(Q_t, Q_g), F_g(Q_t, Q_g)$ and p_{bhp}

 $ightarrow\,$ four equations and four unknowns per standard well



• one conservation equation per phase α (water, oil, gas) and per segment n:



• one control equation for the top segment, e.g. for a prescribed bottom hole pressure:

$$p_{bhp} - p_{bhp}^{target} = 0$$

• unknowns for each standard well for a three-phase black oil system:

 $Q_t(Q_w, Q_o, Q_g), F_w(Q_t, Q_g), F_g(Q_t, Q_g)$ and p_{bhp}

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• one conservation equation per phase α (water, oil, gas) and per segment n:



• one control equation for the top segment, e.g. for a prescribed bottom hole pressure:

$$p_{bhp} - p_{bhp}^{target} = 0$$

• one equation for all other segments with the pressure relationship to the outlet segment:

 $p_n - p_{\text{outlet of n}} - (\text{pressure drop between segment n and its outlet}) = 0$

• unknowns for each standard well for a three-phase black oil system:

 $Q_t(Q_w, Q_o, Q_g), F_w(Q_t, Q_g), F_g(Q_t, Q_g)$ and p_{bhp}

 $ightarrow\,$ four equations and four unknowns per standard well



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• one equation for all other segments with the pressure relationship to the outlet segment:

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• unknowns for each multisegment well for a three-phase black oil system:

 $Q_{t,n}(Q_{w,n},Q_{o,n},Q_{g,n}), F_{w,n}(Q_{t,n},Q_{g,n}), F_{g,n}(Q_{t,n},Q_{g,n}) \text{ and } p_{bhp}, p_n(n \neq \text{ top segment})$

 $\rightarrow~{\rm four~equations}$ and four unknowns per standard well



• one conservation equation per phase α (water, oil, gas) and per segment n:



• one control equation for the top segment, e.g. for a prescribed bottom hole pressure:

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• unknowns for each multisegment well for a three-phase black oil system:

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 $ightarrow\,$ four equations and four unknowns per segment of a multisegment well

B: Linear system for multisegment wells for 3 phases





- A is a #cells × #cells BCRS-matrix, inner matrices is 3 × 3
- *B_j* is a *#segments* × *#cells* BCRS-matrix, inner matrices are 4 × 3
- C_j is a #cells × #segments BCRS-matrix, inner matrices are 3 × 4
- *B_j* and *C_j* contain entries for the cells that are perforated by the well *j*
- B_j and C_j^T have the same sparsity pattern
- standard wells: one row, all entries are in that one row
- multisegment wells: #segments rows, each row has as many entries as the segment has perforations

B: Linear system for multisegment wells for 3 phases



- A is a #cells × #cells BCRS-matrix, inner matrices is 3 × 3
- *B_j* is a *#segments* × *#cells* BCRS-matrix, inner matrices are 4 × 3
- C_j is a #cells × #segments BCRS-matrix, inner matrices are 3 × 4
- *B_j* and *C_j* contain entries for the cells that are perforated by the well *j*
- B_j and C_j^T have the same sparsity pattern
- standard wells: one row, all entries are in that one row
- multisegment wells: #segments rows, each row has as many entries as the segment has perforations

B: Linear system for multisegment wells for 3 phases





- D_j is a #segments × #segments
 BCRS-matrix, inner matrices are 4 × 4
- *D_j* contains entries on the diagonal and for all connections of the multisegment well
- *D_j* is generally **not** symmetric, only the sparsity pattern is symmetric

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- Each process knows only its own parts of *B_j* and *C_j*, no overlap!
- D_j , x_{w_j} , and r_{w_j} are shared by all processes linked to the same well.
- Assembling B_j, C_j, D_j, and r_{w_j} requires communication, since entries depend on all perforations (e.g., pressure differences, averages, maxima, ...).
- One communicator per well!





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Recall the Schur complement approach:

• For each x_{w_i} , solve the small system

$$x_{w_j} = D_j^{-1} \left(r_{w_j} - B_j x_r \right).$$

• For x_r, iteratively solve

$$\left(\mathsf{A} - \sum_{j=1}^{\# wells} C_j D_j^{-1} B_j\right) \mathbf{x}_r = r_r - C_j D_j^{-1} \mathbf{r}_{w_j}.$$





Recall the Schur complement approach:

• For each x_{w_i} , solve the small system

$$x_{w_j} = D_j^{-1} \left(r_{w_j} - B_j x_r \right).$$

• For x_r, iteratively solve

$$A \cdot x_r - \sum_{j=1}^{\# wells} C_j D_j^{-1} B_j \cdot x_r = r_r - C_j D_j^{-1} r_{w_j}.$$





Recall the Schur complement approach:

• For each x_{w_i} , solve the small system

$$x_{w_j} = D_j^{-1} \left(r_{w_j} - \sum_{k=1}^{\text{all ranks}} (B_j)_{|k} (x_r)_{|k} \right).$$

• For x_r, iteratively solve

$$A_{|k} \cdot (x_{r})_{|k} - \sum_{j=1}^{\text{#wells}} (C_{j})_{|k} D_{j}^{-1} \left(\sum_{k=1}^{\text{of well } j} (B_{j})_{|k} (x_{r})_{|k} \right) = (r_{r})_{|k} - \sum_{j=1}^{\text{#wells}} (C_{j})_{|k} D_{j}^{-1} r_{w_{j}}.$$



$$x_{w_{j}} = D_{j}^{-1} \left(r_{w_{j}} - \sum_{k=1}^{\text{all ranks}} (B_{j})_{|k}(x_{r})_{|k} \right) \quad \left| A_{|k} \cdot (x_{r})_{|k} - \sum_{j=1}^{\text{#wells}} (C_{j})_{|k} D_{j}^{-1} \left(\sum_{k=1}^{\text{all ranks}} (B_{j})_{|k}(x_{r})_{|k} \right) = (r_{r})_{|k} - \sum_{j=1}^{\text{#wells}} (C_{j})_{|k} D_{j}^{-1} r_{w_{j}} \right)$$

Where is communication needed when multiplying with B, D^{-1}, C ?

C: Multiplying with B





\rightarrow Communicate to get the full vector $B \cdot x_r$ on all ranks perforated by that well!

C: Multiplying with D^{-1}



$$x_{w_{j}} = D_{j}^{-1} \left(r_{w_{j}} - \sum_{k=1}^{\text{all ranks}} (B_{j})_{|k}(x_{r})_{|k} \right) \quad \left| A_{|k} \cdot (x_{r})_{|k} - \sum_{j=1}^{\text{#wells}} (C_{j})_{|k} D_{j}^{-1} \left(\sum_{k=1}^{\text{all ranks}} (B_{j})_{|k}(x_{r})_{|k} \right) = (r_{r})_{|k} - \sum_{j=1}^{\text{#wells}} (C_{j})_{|k} D_{j}^{-1} r_{w_{j}} \left((B_{j})_{|k} (X_{r})_{|k} \right) = (r_{r})_{|k} - \sum_{j=1}^{\text{#wells}} (C_{j})_{|k} D_{j}^{-1} r_{w_{j}} \left((B_{j})_{|k} (X_{r})_{|k} \right) = (r_{r})_{|k} - \sum_{j=1}^{\text{#wells}} (C_{j})_{|k} D_{j}^{-1} r_{w_{j}} \left((B_{j})_{|k} (X_{r})_{|k} \right) = (r_{r})_{|k} - \sum_{j=1}^{\text{#wells}} (C_{j})_{|k} D_{j}^{-1} r_{w_{j}} \left((B_{j})_{|k} (X_{r})_{|k} \right) = (r_{r})_{|k} - \sum_{j=1}^{\text{#wells}} (C_{j})_{|k} D_{j}^{-1} r_{w_{j}} \left((B_{j})_{|k} (X_{r})_{|k} \right) = (r_{r})_{|k} - \sum_{j=1}^{\text{#wells}} (C_{j})_{|k} D_{j}^{-1} r_{w_{j}} \left((B_{j})_{|k} (X_{r})_{|k} \right) = (r_{r})_{|k} - \sum_{j=1}^{\text{#wells}} (C_{j})_{|k} D_{j}^{-1} r_{w_{j}} \left((B_{j})_{|k} (X_{r})_{|k} \right) = (r_{r})_{|k} - \sum_{j=1}^{\text{#wells}} (C_{j})_{|k} D_{j}^{-1} r_{w_{j}} \left((B_{j})_{|k} (X_{r})_{|k} \right) = (r_{r})_{|k} - \sum_{j=1}^{\text{#wells}} (C_{j})_{|k} D_{j}^{-1} r_{w_{j}} \left((B_{j})_{|k} (X_{r})_{|k} \right) = (r_{r})_{|k} - \sum_{j=1}^{\text{#wells}} (C_{j})_{|k} D_{j}^{-1} r_{w_{j}} \left((B_{j})_{|k} (X_{r})_{|k} \right) = (r_{r})_{|k} - \sum_{j=1}^{\text{#wells}} (C_{j})_{|k} D_{j}^{-1} r_{w_{j}} \left((B_{j})_{|k} (X_{r})_{|k} \right) = (r_{r})_{|k} - \sum_{j=1}^{\text{#wells}} (C_{j})_{|k} D_{j}^{-1} r_{w_{j}} \left((B_{j})_{|k} (X_{r})_{|k} \right) = (r_{r})_{|k} - \sum_{j=1}^{\text{#wells}} (C_{j})_{|k} D_{j}^{-1} r_{w_{j}} \left((B_{j})_{|k} (X_{r})_{|k} \right) = (r_{r})_{|k} - \sum_{j=1}^{\text{#wells}} (C_{j})_{|k} D_{j}^{-1} r_{w_{j}} \left((B_{j})_{|k} (X_{r})_{|k} \right) = (r_{r})_{|k} - \sum_{j=1}^{\text{#wells}} (C_{j})_{|k} D_{j}^{-1} r_{w_{j}} \left((B_{j})_{|k} (X_{r})_{|k} \right) = (r_{r})_{|k} - \sum_{j=1}^{\text{#wells}} (C_{j})_{|k} (X_{r})_{|k} \left((B_{j})_{|k} (X_{r})_{|k} (X_{r})_{|k} \right) = (r_{r})_{|k} - \sum_{j=1}^{\text{#wells}} (C_{j})_{|k} (X_{r})_{|k} \left((B_{j})_{|k} (X_{r})_{|k} (X_{r})_{|k} \right) = (r_{r})_{|k} (X_{r})_{|k} (X_{r})_{|k} (X_{r})_{|k} \left((B_{j})_{|$$

→ Calculate $D_j^{-1} \cdot (B \cdot x_r)$ and $D_j^{-1} \cdot r_{w_j}$ on each rank! → No communication needed, because D_j , $(B \cdot x_r)$ and r_{w_j} are the same across all ranks of well j!

C: Multiplying with C





 \rightarrow Multiplication with $(C_i)_{|k}$ will automatically reduce to rank k!

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D: opm-tests/norne/NORNE_ATW2013.DATA





- 8|16 procs, --allow-distributed-wells=true|false --partition-method=zoltan|zoltanwell --allow-splitting-inactive-wells=true - Run on a machine with 32 cores, 64 threads, 4.55 GHz max frequency, 128 GB RAM.

D: opm-tests/norne/NORNE_ATW2013.DATA





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D: Case with pprox 1.64 M active cells, 338 active wells





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E: Outlook



- Currently works for several test cases, but not for all \odot !
- More thorough verification of correctness needed!
- General problem with distriuted wells:

| Load balancing | distribu | ited the | wells as | s follows: |
|----------------|----------|----------|-----------|--------------|
| well name | | ranks w: | ith perfo | orated cells |
| | | | | |
| PR0D1 | . 0 |) : | 1 | |
| PR0D2 | e |) : | 1 : | 2 |
| PROD3 | e |) : | 1 : | 2 |
| | | | | |

- $\rightarrow~$ Careful with throws inside a loop over wells!
- Next steps: alternative solvers or preconditioners for the well equations?
- Many thanks to Markus Blatt, Michal Tóth, Vegard Kippe, Kai Bao and Halvor Møll Nilsen! \heartsuit