

New CPR based solver for OPM Flow

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Motivation

- / Multisegment Well contributions
- / Well Solver flexibility
- / Linear System with mixed physics

The diagram illustrates a linear system with mixed physics. The matrix is partitioned into blocks: a large square block A, two vertical blocks C1 and C2, two horizontal blocks B1 and B2, and two small square blocks D1 and D2. This matrix is multiplied by a vector Xr, which is partitioned into a large block Xr and two smaller blocks X1 and X2. The result is a vector Rr, which is partitioned into a large block Rr and two smaller blocks R1 and R2.

$$\begin{pmatrix} A & C1 & C2 \\ B1 & D1 & \\ B2 & & D2 \end{pmatrix} \times \begin{pmatrix} Xr \\ X1 \\ X2 \end{pmatrix} = \begin{pmatrix} Rr \\ R1 \\ R2 \end{pmatrix}$$

Motivation

Well contributions can be applied in two ways

$$\hat{A} = A - CD^{-1}B$$

- CPR applied to \tilde{A} ;
- D^{-1} built at assembly.



CPR
Cost



Iterations
number

$$Ax_r - CD^{-1}Bx_r$$

- CPR applied to A ;
- $Dz=Bx_r$ solved per iteration.



CPR
Cost



Iterations
number

System CPR Preconditioner

- / 4 Stages preconditioner
- / Blocks with clear physical meaning
- / Flexibility on wells solver
- / Schur complement free

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} x_r \\ x_w \end{bmatrix} = \begin{bmatrix} d_r \\ d_w \end{bmatrix}$$

$$Ax_r + Bx_w = b_r$$

$$Cx_r + Dx_w = b_w$$

$$x_r^{k+1} = x_r^k + \delta x_r^{k+1/2} + \Delta x_r^{k+1/2}$$

$$x_w^{k+1} = x_w^k + \delta x_w^{k+1/2} + \Delta x_w^{k+1/2}$$

System CPR Preconditioner

The Krylov subspaces

- GMRes - 1 preconditioner call
- FlexGMRes - 1 preconditioner call
- BicGStab - 2 preconditioner calls

$$z_j = M^{-1}v_j \} \text{Arnoldi base vectors}$$

$$\hat{p}_k = M^{-1}p_k \} \text{Search directions}$$

System CPR Preconditioner

Preconditioner apply method

Stage 1: Reservoir Solver

$$\begin{aligned}\delta z_r &= A_{CPR}^{-1} v_r , \\ z_r &\leftarrow z_r + \delta z_r , \\ v_r &\leftarrow v_r - A \delta z_r , \\ v_w &\leftarrow v_w - C \delta z_r .\end{aligned}$$

Stage 3: Reservoir smoothing

$$\begin{aligned}\delta z_r &= A_{ILU0}^{-1} v_r , \\ z_r &\leftarrow z_r + \delta z_r , \\ v_w &\leftarrow v_w - C \delta z_r .\end{aligned}$$

Stage 2: Well solver

$$\begin{aligned}\delta z_w &= D^{-1} v_w , \\ z_w &\leftarrow z_w + \delta z_w , \\ v_r &\leftarrow v_r - B \delta z_w , \\ v_w &\leftarrow v_w - D \delta z_w .\end{aligned}$$

Stage 4: Well Solver

$$\begin{aligned}\delta z_w &= D^{-1} v_w , \\ z_w &\leftarrow z_w + \delta z_w .\end{aligned}$$

Early Results

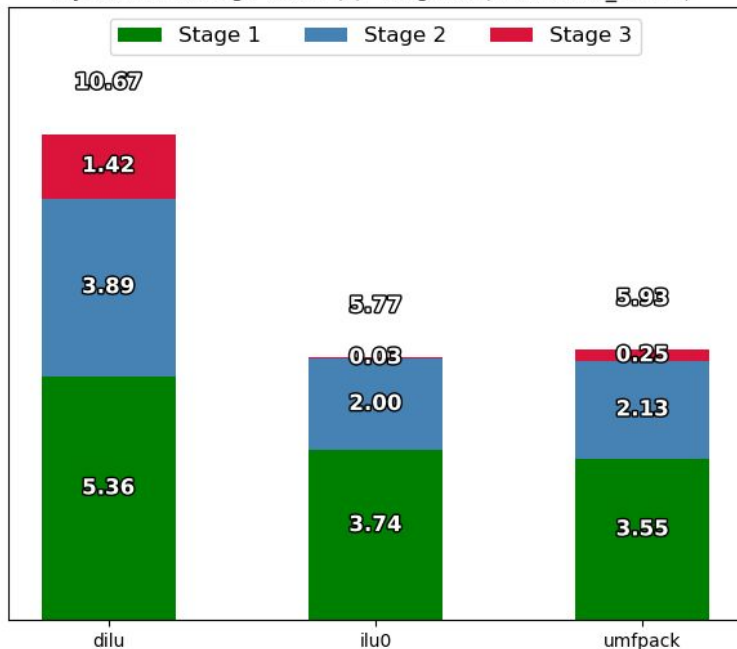
Machine

- CPU: AMD Ryzen 9 7900
- GPU: AMD Radeon PRO W7900

```
"maxiter": "20",  
"tol": "1e-3",  
"use_well_weights": "false",  
"add_wells": "false",  
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Early Results

System CPR Stage Times (s) - bicgstab (SECTOR01_MS1)

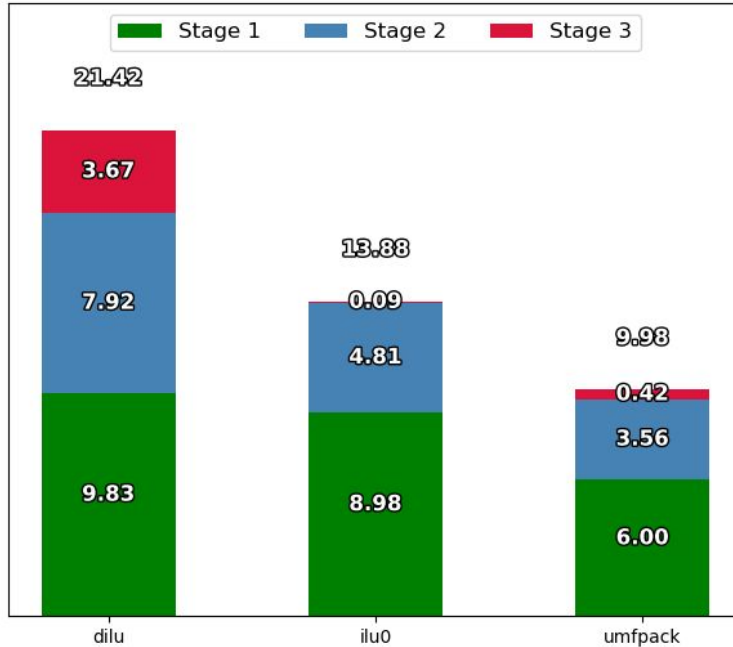


BiCGStab — SECTOR01_MS1

	Simulation Time	Linear Solver Time	Newton Iterations	Linear Iterations
default	16.00 s	7.62 s	510 (0)	859 (0)
dilu	35.23 s	18.52 s	680 (53)	1052 (74)
ilu0	20.18 s	10.91 s	576 (0)	980 (0)
umfpack	21.41 s	11.40 s	632 (21)	888 (2)

Early Results

System CPR Stage Times (s) - gmres (SECTOR01_MS1)

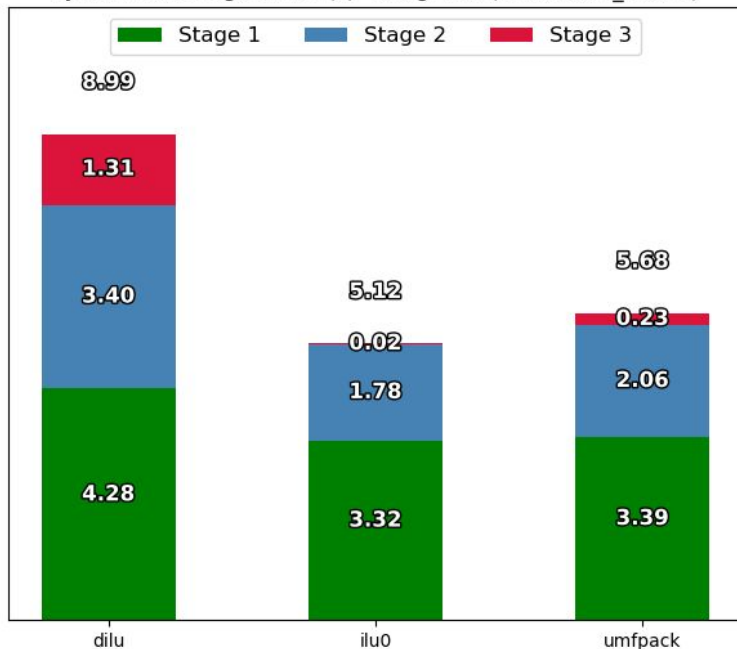


GMRes — SECTOR01_MS1

	Simulation Time	Linear Solver Time	Newton Iterations	Linear Iterations
default	14.79 s	8.52 s	359 (0)	2885 (0)
dilu	41.31 s	28.44 s	471 (51)	4239 (652)
ilu0	30.70 s	20.74 s	602 (157)	5062 (1324)
umfpack	21.26 s	14.52 s	402 (42)	3327 (271)

Early Results

System CPR Stage Times (s) - flexgmres (SECTOR01_MS01)

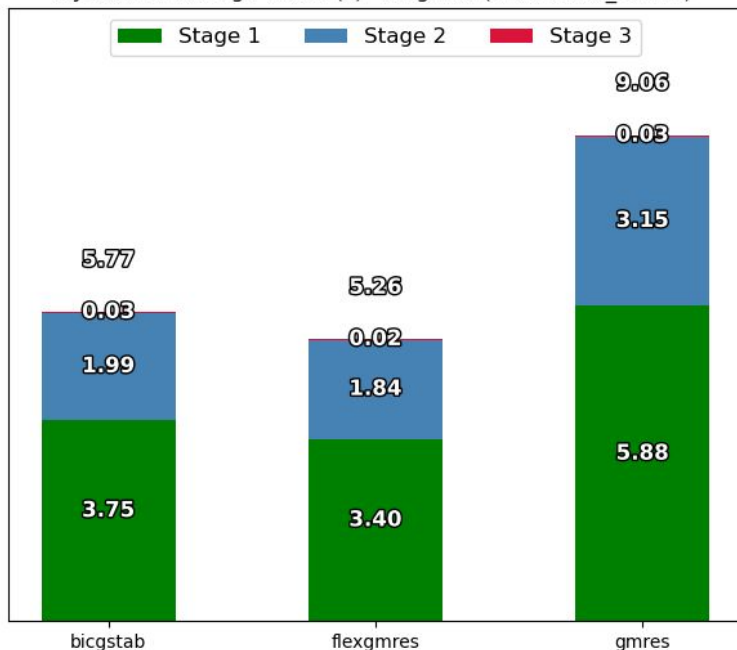


FlexGMRes — SECTOR01_MS01

	Simulation Time	Linear Solver Time	Newton Iterations	Linear Iterations
default	16.49 s	7.86 s	532 (32)	1777 (107)
dilu	31.33 s	16.32 s	602 (21)	1978 (53)
ilu0	21.07 s	10.98 s	624 (0)	2018 (0)
umfpack	22.10 s	11.76 s	652 (42)	2047 (160)

Early Results

System CPR Stage Times (s) - bicgstab (SECTOR01_MS1)



BiCGStab — SECTOR01_MS1

	Simulation Time	Linear Solver Time	Newton Iterations	Linear Iterations
default	15.85 s	7.60 s	510 (0)	859 (0)
bicgstab	20.04 s	10.87 s	576 (0)	980 (0)

GMRes — SECTOR01_MS1

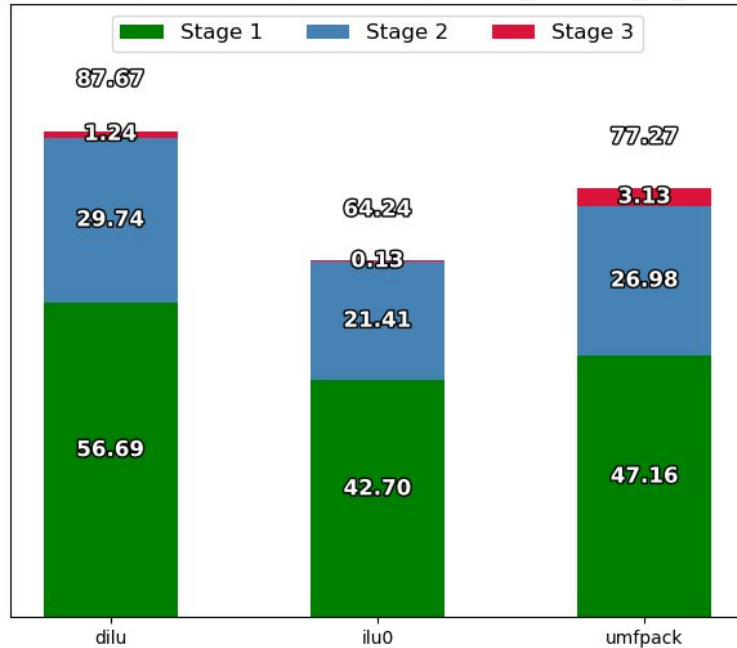
	Simulation Time	Linear Solver Time	Newton Iterations	Linear Iterations
default	14.74 s	8.52 s	359 (0)	2885 (0)
gmres	20.40 s	13.62 s	402 (42)	3327 (271)

FlexGMRes — SECTOR01_MS1

	Simulation Time	Linear Solver Time	Newton Iterations	Linear Iterations
default	16.43 s	7.86 s	532 (32)	1777 (107)
flexgmres	22.10 s	11.49 s	671 (42)	2076 (153)

Early Results

System CPR Stage Times (s) - bicgstab (NORNE_ATW2013_1A_MSW)

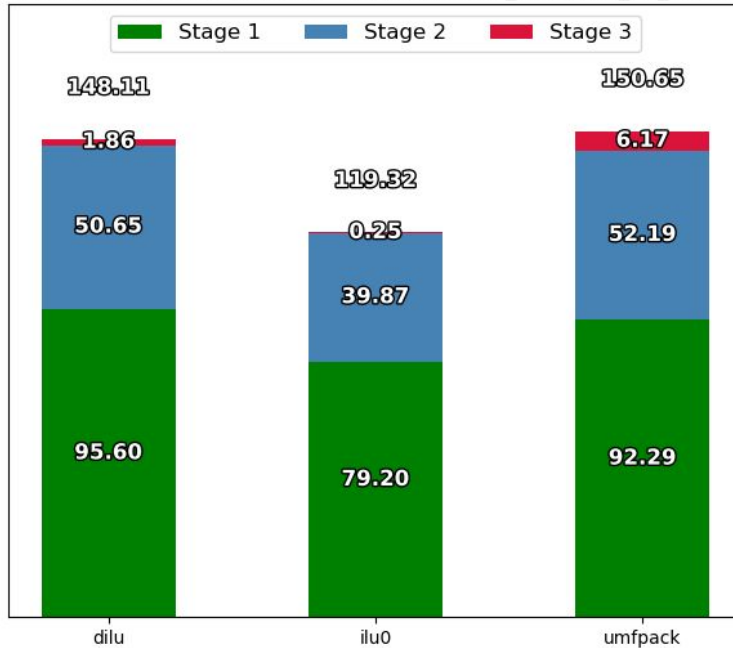


BiCGStab — NORNE_ATW2013_1A_MSW

	Simulation Time	Linear Solver Time	Newton Iterations	Linear Iterations
default	271.07 s	143.61 s	1395 (184)	7418 (1582)
dilu	577.00 s	198.58 s	3549 (357)	2039 (311)
ilu0	439.72 s	155.96 s	3541 (189)	1748 (75)
umfpack	446.48 s	168.20 s	3486 (252)	2217 (164)

Early Results

System CPR Stage Times (s) - gmres (NORNE_ATW2013_1A_MSW)

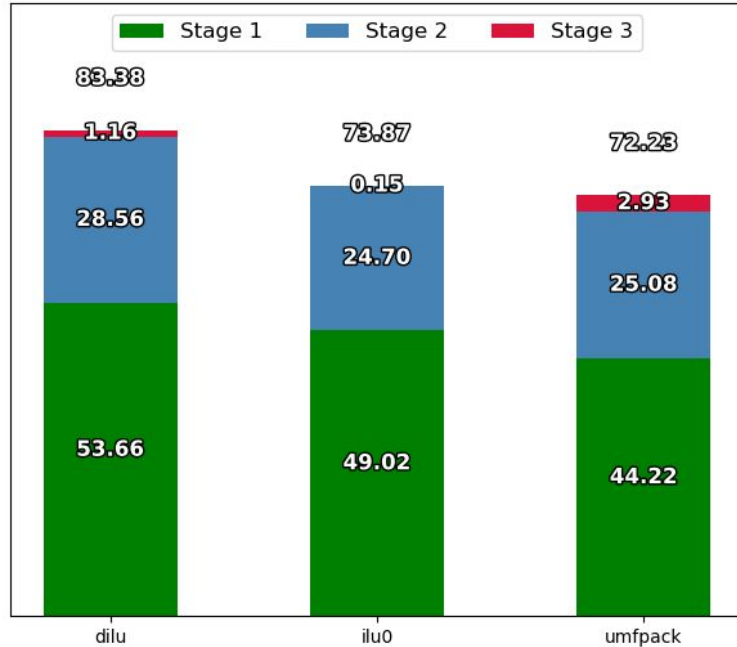


GMRes — NORNE_ATW2013_1A_MSW

	Simulation Time	Linear Solver Time	Newton Iterations	Linear Iterations
default	238.28 s	120.91 s	1268 (152)	10981 (1973)
dilu	344.63 s	202.32 s	1169 (70)	10761 (1018)
ilu0	273.48 s	164.91 s	1157 (132)	10728 (1788)
umfpack	322.79 s	203.56 s	1317 (109)	12372 (1500)

Early Results

System CPR Stage Times (s) - flexgmres (NORNE_ATW2013_1A_MSW)

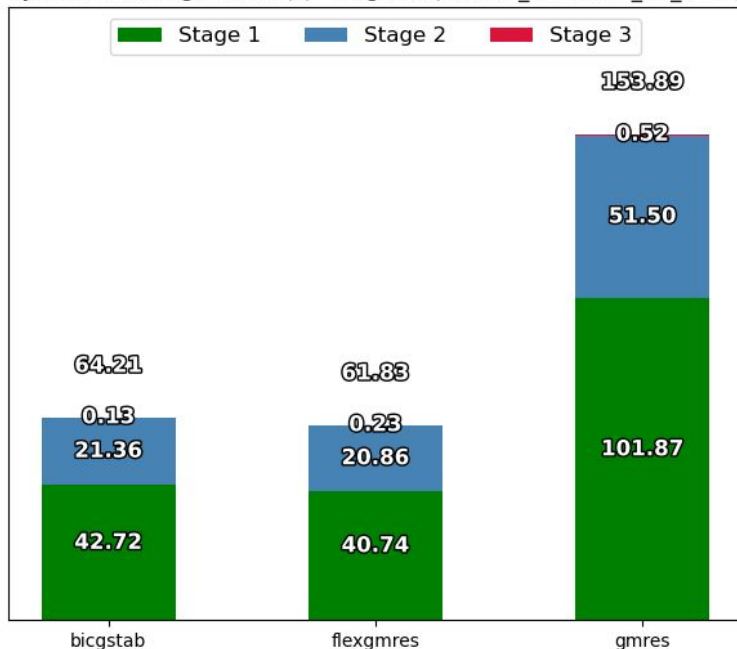


FlexGMRes — NORNE_ATW2013_1A_MSW

	Simulation Time	Linear Solver Time	Newton Iterations	Linear Iterations
default	227.48 s	102.31 s	1381 (77)	9063 (875)
dilu	575.93 s	199.57 s	3536 (378)	6753 (745)
ilu0	582.25 s	204.23 s	4808 (420)	7451 (677)
umfpack	445.21 s	167.10 s	3421 (462)	6693 (894)

Early Results

System CPR Stage Times (s) - bicgstab (NORNE_ATW2013_1A_MSW)



BiCGStab – NORNE_ATW2013_1A_MSW

	Simulation Time	Linear Solver Time	Newton Iterations	Linear Iterations
default	269.50 s	142.62 s	1395 (184)	7418 (1582)
bicgstab	437.08 s	155.93 s	3541 (189)	1748 (75)

GMRes – NORNE_ATW2013_1A_MSW

	Simulation Time	Linear Solver Time	Newton Iterations	Linear Iterations
default	237.67 s	120.76 s	1268 (152)	10981 (1973)
gmres	341.67 s	212.01 s	1468 (216)	13746 (2955)

FlexGMRes – NORNE_ATW2013_1A_MSW

	Simulation Time	Linear Solver Time	Newton Iterations	Linear Iterations
default	226.79 s	102.09 s	1381 (77)	9063 (875)
flexgmres	424.15 s	154.07 s	3372 (273)	6244 (631)

Next Steps

- / GPU implementation
- / Measure the impact on bigger models. More wells and more segments.
- / Compositional model
- / Thermal model

Acknowledgements



Motivation

For `=true` the implications to CPR are

- Additional fill-in in the matrix -> higher CPR cost
- Better convergence, preconditioner accounts for well-cell coupling

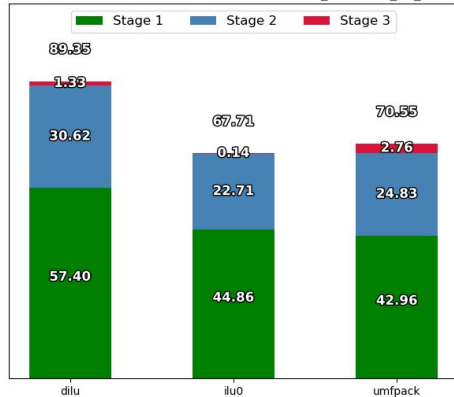
For `=false` the implications to CPR are

- Lower CPR cost
- Less accurate approximation of \tilde{A}
- More iterations

System CPR Preconditioner

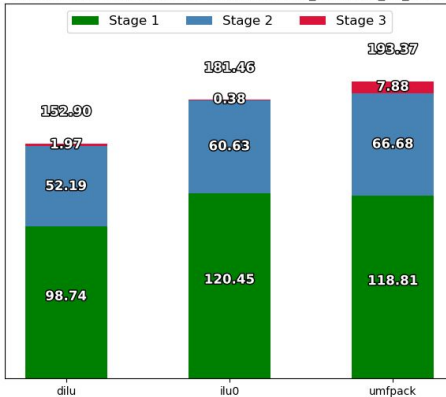
Standard CPR extracts only the pressure from the coupled system. Block architecture of System CPR is the right structural foundation for extending preconditioners to composital and thermal models. For thermal and compositional: the system must be partitioned into explicit blocks, each handled by an appropriate sub-solver.

System CPR Stage Times (s) - bicgstab (NORNE_ATW2013_1B_MSW)

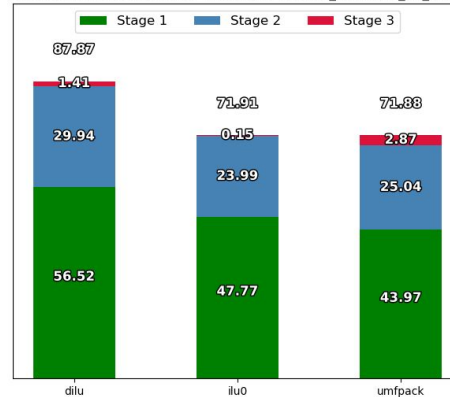
**BiCGStab — NORNE_ATW2013_1B_MSW**

	Simulation Time	Linear Solver Time	Newton Iterations	Linear Iterations
default	325.74 s	165.51 s	1802 (357)	8183 (1188)
dilu	688.22 s	223.49 s	4379 (420)	1801 (281)
ilu0	524.23 s	178.85 s	4346 (273)	1618 (205)
umfpack	461.40 s	166.79 s	3768 (378)	1724 (326)

System CPR Stage Times (s) - gmres (NORNE_ATW2013_1B_MSW)



System CPR Stage Times (s) - flexgmres (NORNE_ATW2013_1B_MSW)

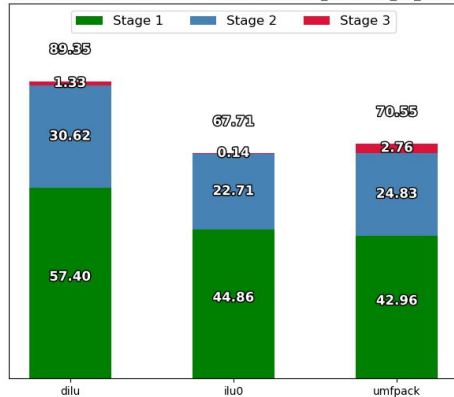
**GMRes — NORNE_ATW2013_1B_MSW**

	Simulation Time	Linear Solver Time	Newton Iterations	Linear Iterations
default	444.36 s	215.78 s	2691 (336)	17551 (1847)
dilu	387.91 s	214.59 s	1410 (400)	10842 (2658)
ilu0	443.01 s	258.54 s	2131 (483)	15926 (2708)
umfpack	432.93 s	266.10 s	1908 (414)	15512 (3223)

FlexGMRes — NORNE_ATW2013_1B_MSW

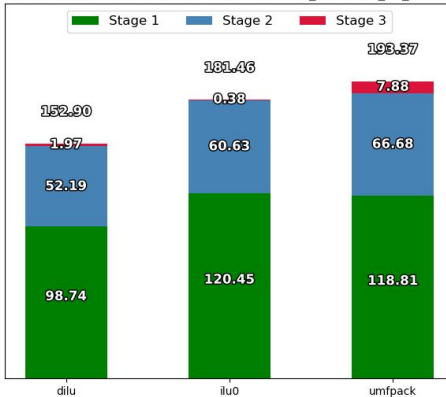
	Simulation Time	Linear Solver Time	Newton Iterations	Linear Iterations
default	310.54 s	142.07 s	1881 (361)	12599 (1924)
dilu	780.34 s	248.56 s	4987 (630)	7059 (993)
ilu0	608.89 s	208.29 s	5050 (504)	7232 (1014)
umfpack	578.01 s	199.73 s	4699 (357)	6657 (628)

System CPR Stage Times (s) - bicgstab (NORNE_ATW2013_1B_MSW)

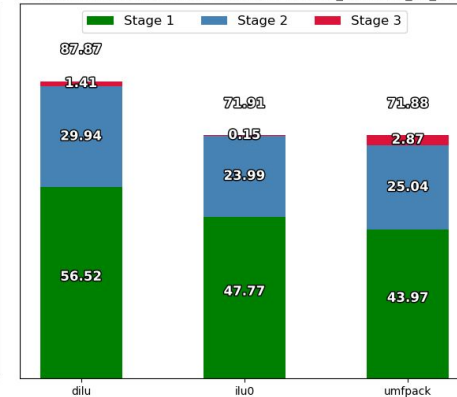
**BiCGStab — NORNE_ATW2013_1B_MSW**

	Simulation Time	Linear Solver Time	Newton Iterations	Linear Iterations
default	325.74 s	165.51 s	1802 (357)	8183 (1188)
dilu	688.22 s	223.49 s	4379 (420)	1801 (281)
ilu0	524.23 s	178.85 s	4346 (273)	1618 (205)
umfpack	461.40 s	166.79 s	3768 (378)	1724 (326)

System CPR Stage Times (s) - gmres (NORNE_ATW2013_1B_MSW)



System CPR Stage Times (s) - flexgmres (NORNE_ATW2013_1B_MSW)

**GMRes — NORNE_ATW2013_1B_MSW**

	Simulation Time	Linear Solver Time	Newton Iterations	Linear Iterations
default	444.36 s	215.78 s	2691 (336)	17551 (1847)
dilu	387.91 s	214.59 s	1410 (400)	10842 (2658)
ilu0	443.01 s	258.54 s	2131 (483)	15926 (2708)
umfpack	432.93 s	266.10 s	1908 (414)	15512 (3223)

FlexGMRes — NORNE_ATW2013_1B_MSW

	Simulation Time	Linear Solver Time	Newton Iterations	Linear Iterations
default	310.54 s	142.07 s	1881 (361)	12599 (1924)
dilu	780.34 s	248.56 s	4987 (630)	7059 (993)
ilu0	608.89 s	208.29 s	5050 (504)	7232 (1014)
umfpack	578.01 s	199.73 s	4699 (357)	6657 (628)

BiCGStab — NORNE_ATW2013_2A_MS

	Simulation Time	Linear Solver Time	Newton Iterations	Linear Iterations
default	123.07 s	49.99 s	675 (168)	2265 (346)
ilu0	188.33 s	75.35 s	1220 (168)	1614 (206)
umfpack	199.06 s	80.17 s	1309 (168)	1597 (165)

BiCGStab — NORNE_ATW2013_2A_MS

	Simulation Time	Linear Solver Time	Newton Iterations	Linear Iterations
default	122.69 s	49.83 s	675 (168)	2265 (346)
bicgstab	186.31 s	74.59 s	1220 (168)	1614 (206)

GMRes — NORNE_ATW2013_2A_MS

	Simulation Time	Linear Solver Time	Newton Iterations	Linear Iterations
default	129.11 s	53.52 s	689 (180)	4474 (839)
dilu	229.81 s	109.34 s	877 (315)	5123 (1334)
ilu0	145.86 s	74.73 s	640 (147)	4429 (844)
umfpack	157.67 s	81.56 s	701 (210)	4546 (999)

GMRes — NORNE_ATW2013_2A_MS

	Simulation Time	Linear Solver Time	Newton Iterations	Linear Iterations
default	129.04 s	53.60 s	689 (180)	4474 (839)
gmres	170.08 s	85.38 s	811 (273)	4889 (1194)

FlexGMRes — NORNE_ATW2013_2A_MS

	Simulation Time	Linear Solver Time	Newton Iterations	Linear Iterations
default	122.22 s	46.24 s	694 (168)	3915 (658)
dilu	249.44 s	89.29 s	1343 (168)	3470 (393)
ilu0	195.21 s	73.21 s	1315 (168)	3449 (390)
umfpack	201.52 s	77.70 s	1382 (231)	3539 (473)

FlexGMRes — NORNE_ATW2013_2A_MS

	Simulation Time	Linear Solver Time	Newton Iterations	Linear Iterations
default	121.16 s	46.28 s	694 (168)	3915 (658)
flexgmres	207.05 s	79.43 s	1417 (231)	3777 (574)

BiCGStab – NORNE_ATW2013_3A_MSW

	Simulation Time	Linear Solver Time	Newton Iterations	Linear Iterations
default	129.01 s	56.46 s	680 (21)	2717 (84)
ilu0	189.62 s	82.06 s	1148 (21)	2063 (6)
umfpack	192.18 s	85.63 s	1143 (21)	2088 (7)

BiCGStab – NORNE_ATW2013_3A_MSW

	Simulation Time	Linear Solver Time	Newton Iterations	Linear Iterations
default	129.59 s	56.50 s	680 (21)	2717 (84)
bicgstab	189.81 s	82.17 s	1148 (21)	2063 (6)

GMRes – NORNE_ATW2013_3A_MSW

	Simulation Time	Linear Solver Time	Newton Iterations	Linear Iterations
default	159.87 s	74.94 s	831 (147)	6780 (1124)
ilu0	200.76 s	113.92 s	878 (168)	7155 (1295)
umfpack	178.96 s	102.39 s	737 (42)	6139 (324)

GMRes – NORNE_ATW2013_3A_MSW

	Simulation Time	Linear Solver Time	Newton Iterations	Linear Iterations
default	159.73 s	75.02 s	831 (147)	6780 (1124)
gmres	191.45 s	108.24 s	827 (147)	6757 (1094)

FlexGMRes – NORNE_ATW2013_3A_MSW

	Simulation Time	Linear Solver Time	Newton Iterations	Linear Iterations
default	133.88 s	53.78 s	729 (21)	4806 (153)
ilu0	186.97 s	78.45 s	1183 (21)	4202 (41)
umfpack	189.31 s	80.10 s	1183 (21)	4109 (104)

FlexGMRes – NORNE_ATW2013_3A_MSW

	Simulation Time	Linear Solver Time	Newton Iterations	Linear Iterations
default	130.91 s	53.56 s	729 (21)	4806 (153)
flexgmres	175.47 s	73.33 s	1092 (0)	3912 (0)