Simulating fluid flow in fractured reservoirs with The Matlab Reservoir Toolbox (MRST)

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Agenda

› Modeling philosophy
› Equations and discretizations
› Modeling concepts for fractures
› Numerical examples
Mathematical models

› Darcy’s law (unit viscosity):

\[ v = - K \nabla p \]

› Conservation of mass (single phase, incompressible) \( \rightarrow \)

\[ \nabla \cdot v = q \]

› Pressure equation:

\[ -\nabla \cdot (K \nabla p) = q \]

› Tracer transport:

\[ \phi \frac{\partial c}{\partial t} + \nabla \cdot (cv) = q_t \]
Finite-volume method

› Integral formulation
\[ \int_{\partial \Omega_i} (\mathbf{v} \cdot \mathbf{n}) \, dS = \int_{\Omega_i} q \, d\Omega \]

› Flux approximation
\[ \int_S (\mathbf{v} \cdot \mathbf{n}) \, dS \approx \sum_{k=1}^{\nu} t_k \rho_k \]

› Linear system
\[ A \mathbf{p} = \mathbf{q} \]
Modeling concepts

Hierarchical modeling:

• Large-scale fractures: explicitly
• Small-scale fractures: effective permeability (anisotropic)
• Consistent and robust discretization of flow in fractured reservoirs
  › incompTPFA_DFM / incompMPFA_DFM
But…..

• What if the conditions for upscaling is not valid? (no scale – separation etc.)
• Only coarse transport

We want a method that is:

1. Comparable with upscaling when the conditions for upscaling apply
2. Otherwise, comparable with standard fine-scale solvers
3. Capable of fine-scale transport
Modeling concepts

› MsFVM: Multiscale finite volume method (Jenny 2003)
  – Coarse grids
  – Post-processing
  – Direct method / Multiscale method
  – Inexact solver / Preconditioner in GMRES
› solveMSFV_TPFA_Incomp_DFM
Example setup

- Aperture large-scale fractures: 1mm
- Aperture small-scale fractures: 0.5mm
- Fracture permeability = $\frac{\text{aperture}^2}{12}$
- Fracture porosity: 1
- Matrix permeability: 1mDarcy
- Matrix porosity: 0.01
Case 1

› Conforming coarse grid

› Non-conforming coarse grid
Compare as multiscale method
Compare as multiscale method

› Multiscale solution

› Reference solution
Compare as preconditioner

AMG: Y. Notay 2010
Case 2

› Conforming coarse grid

› Non-conforming coarse grid
Case 2

› Direct method

› 10 GMRES iterations
Compare as preconditioners

GMRES residuals

relative residual

0 1 2 3 4 5 6 7 8 9 10
iterations

10^{-15} 10^{-10} 10^{-5} 10^0

Conforming coarse grid
Non-conforming coarse grid
amg
Transport upscaling

› Coarse transport

› Hierarchical models
  • Large-scale fractures explicit

› Dual models
  • Small-scale fractures into a fracture continuum
Transport models

![Graph showing concentration vs PVI for different transport models.]

- Coarse scale transport (5)
- Hierarchical single continuum (10)
- Hierarchical dual model (15)
- Fine-scale transport (12733)
After thoughts

› Pressure equation:
  • Linking uncertainty and stopping criteria for the linear solvers

› Transport
  • Flow based upscaling
  • aMINC (automatic Multiple INteracting Continua)