Higher-Order methods in OPM. Based on the paper ”A Second-Order Finite Volume Method for Field-Scale Reservoir Simulation”

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Introduction

The first-order finite volume (FV) is the default option in many standard reservoir simulators, both commercial and open-source.

+ It is robust.
+ Easy implementation.
– Suffers from numerical diffusion.
– Incorrect computations of the front position, components concentrations, water breakthrough, etc.

Introduction

To reduce the numerical diffusion and increase the accuracy, there are mainly two options:

- to refine the grid,
- to increase the order of the numerical method.

Content of the presentation

1. Second-order method with linear programming reconstruction
2. Explore the method's capabilities in a realistic setting.
   - accuracy
   - verification in the absence of "true" solution
3. Run WAG and $CO2$ injection scenarios on the realistic test cases:
   - a medium-sized realistic reservoir with an unstructured corner point grid
   - an openly available Norne field mode

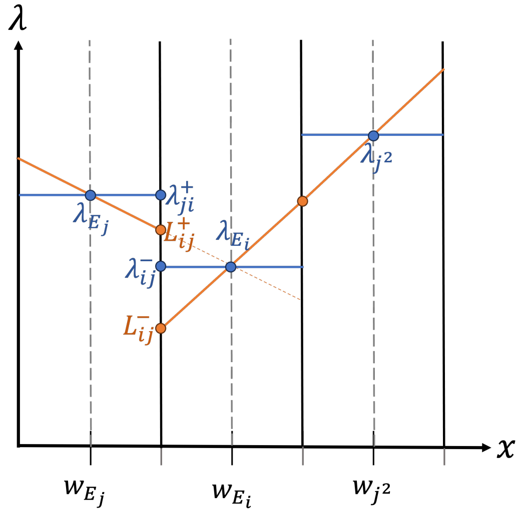
Build and run

The models and the build instructions are available in the repository https://github.com/kvashchuka/second-order-opm-tests.

To run a test case with the second-order method, you need to enable certain flags:

```
./*path_to_the_build_folder_of_opm-simulators*/bin/flow CASE_NAME --enable-higher-order=1 --enable-local-reconstruction=1 --reconstruction-scheme-id=3 --only-reconstruction-for-solvent-or-polymer=false
```

First- vs Second-Order FV Method



First-order FV method

$$\lambda_w^{ij} = \begin{cases} \lambda_{ij}^{-}, & \text{if } \left(\nabla p_w^{n+1} - \rho_w \mathbf{g}\right) \cdot \mathbf{n} \geq 0, \\ \lambda_{ij}^{+}, & \text{otherwise,} \end{cases}$$

Second-order FV method

$$\lambda_w^{ij} = \begin{cases} L_{ij}^{-}, & \text{if } \left(\nabla p_w^{n+1} - \rho_w \mathbf{g}\right) \cdot \mathbf{n} \geq 0, \\ L_{ij}^{+}, & \text{otherwise,} \end{cases}$$

where the linear reconstruction function has to satisfy the following requirements:

$$L_{E_i}(x) := \lambda_{E_i} + \nabla L_{E_i} \cdot (x - \mathbf{w}_{E_i}),$$
$$L_{E_i}(\mathbf{w}_{E_j}) = \lambda_{E_j}, \forall (E_i, E_j) \in \partial E_i.$$

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Second-order method with Linear Programming reconstruction

We want to minimize the total gaps between the reconstructed values and the cell-averaged values at all neighboring cells:

$$\delta(L) := \sum_{\forall (E_i, E_j) \in \partial E_i} |\lambda_{E_j} - L_{E_i}(\mathbf{w}_{E_j})|. \quad (1)$$

The constraints are the following monotonicity conditions:

$$\min\{\lambda_{E_i}, \lambda_{E_j}\} \leq L_{E_i}(\mathbf{w}_{E_j}) \leq \max\{\lambda_{E_i}, \lambda_{E_j}\}, \ \forall (E_i, E_j) \in \partial E_i. \quad (2)$$

Second-order method with Linear Programming reconstruction

We solve the following LP problem:

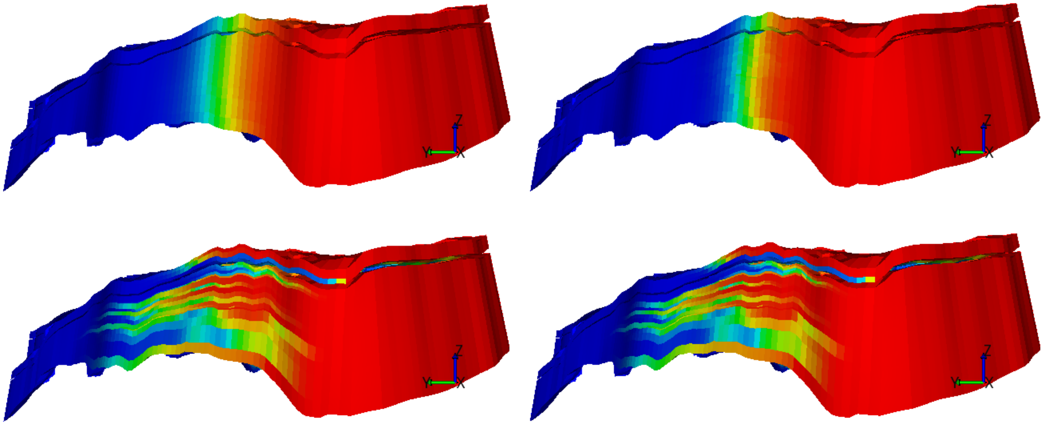
$$\begin{aligned} &\max \sum_{\forall (E_i,E_j)\in\partial E_i} \operatorname{sgn}(v_{E_j})(\mathbf{w}_{E_j} - \mathbf{w}_{E_i}) \cdot \nabla L_{E_i} \\ &\text{subject to } v_{E_j}^- \leq (\mathbf{w}_{E_j} - \mathbf{w}_{E_i}) \cdot \nabla L_{E_i} \leq v_{E_j}^+, \end{aligned} \tag{3}$$

where

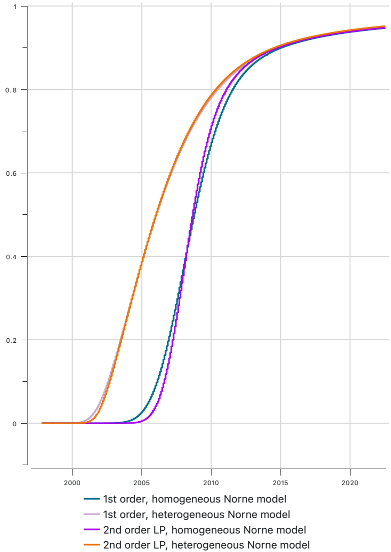
$$\begin{aligned} v_{E_j}^- &= \min\{0, \lambda_{E_i} - \lambda_{E_j}\}, \\ v_{E_j}^+ &= \max\{0, \lambda_{E_i} - \lambda_{E_j}\}. \end{aligned} \tag{4}$$

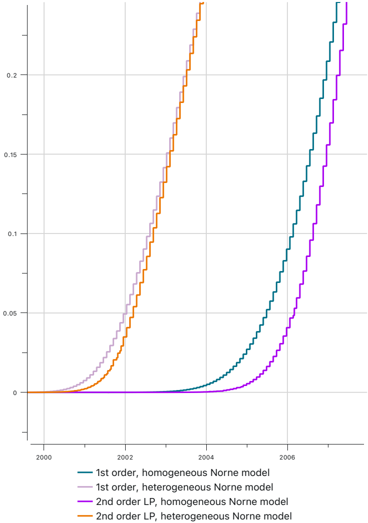
The unknown vector x is the gradient of the linear reconstruction $x = [\nabla L_{E_i}^x, \nabla L_{E_i}^y, \nabla L_{E_i}^z]^T$.
We use an all-inequality simplex method to solve the LP.

Norne: homogeneous (top) and heterogeneous (bottom)

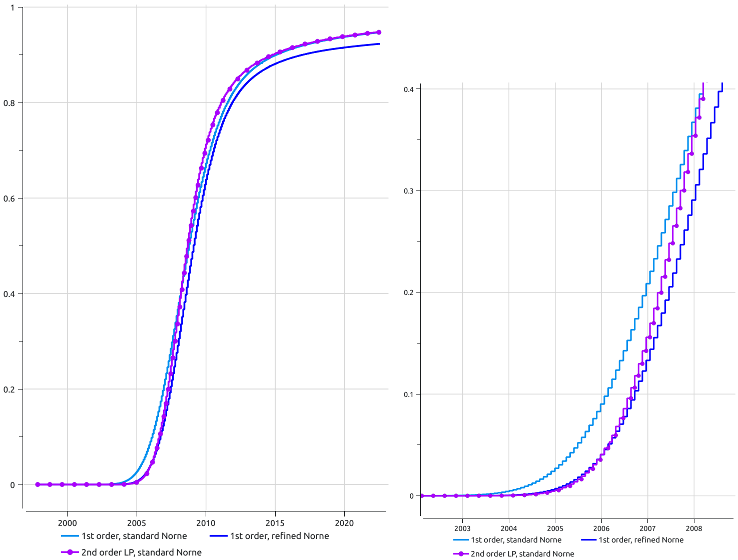


Norne: homogeneous and heterogeneous





Norne: verification with refined model



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CO_2 injection on Norne

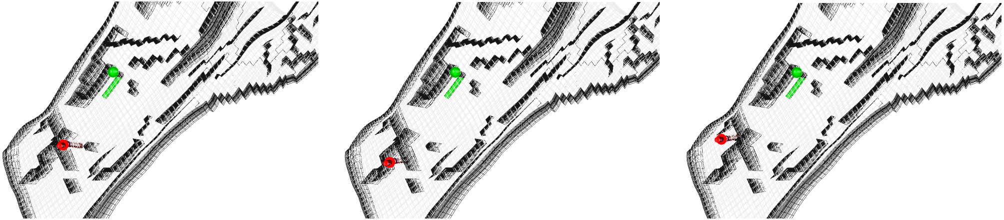


Figure: Positions of the wells in the Norne CO2 injection scenarios: left for the wells in the same compartment, middle - wells are separated by a fault, right - injection well in the corner.

CO_2 injection on Norne

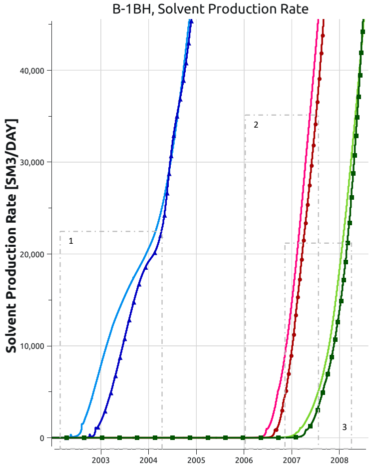


Figure: Solvent production rate for the three scenarios of solvent injection on Norne.

CO_2 injection on Norne: zoom-in

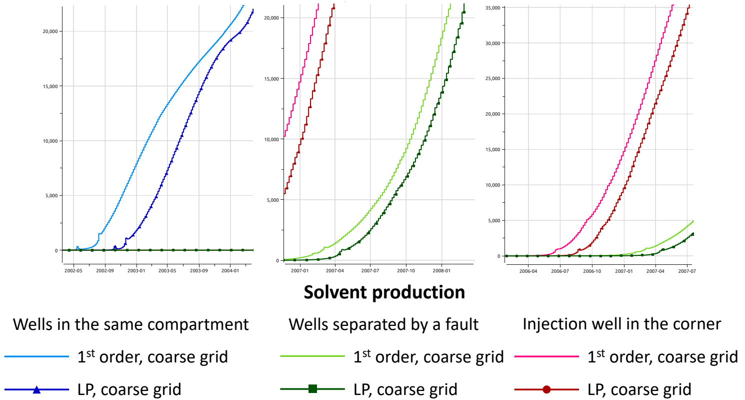
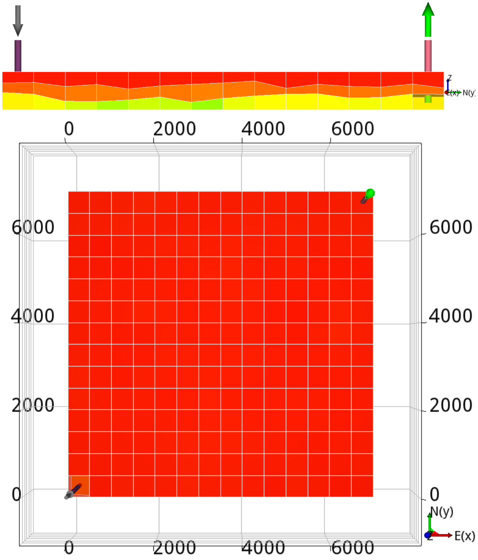
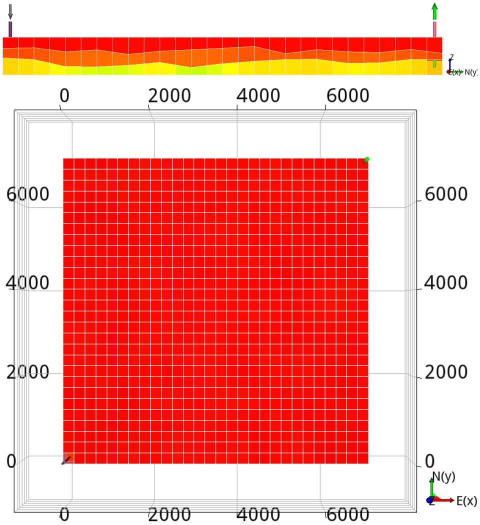


Figure: The subplots zoom into the times of solvent arrival for each scenario: (1) Wells in the same compartment; (2) Wells separated by a fault; (3) Injection well in the corner.

A medium-sized realistic reservoir with an unstructured corner point grid





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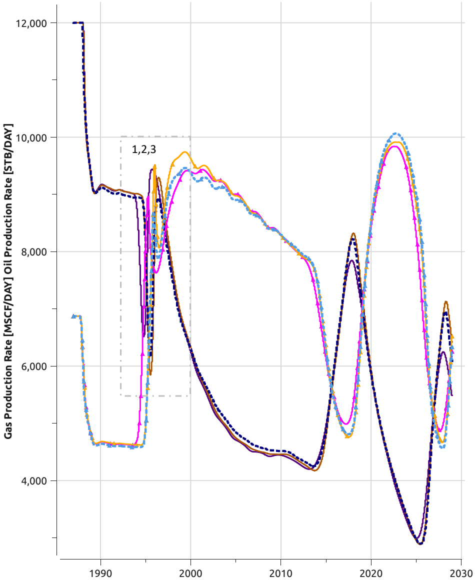
Oil and Gas production rate



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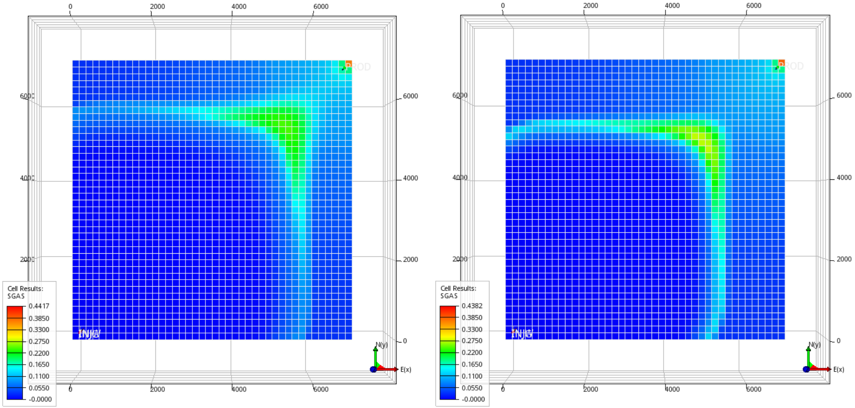
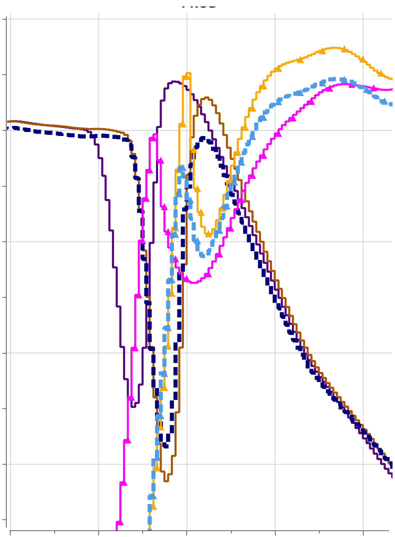
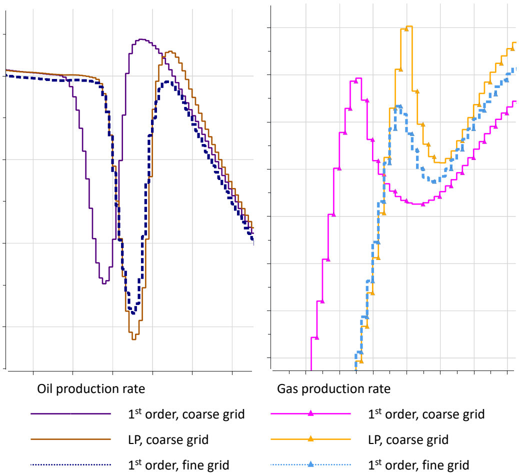
Gas wave

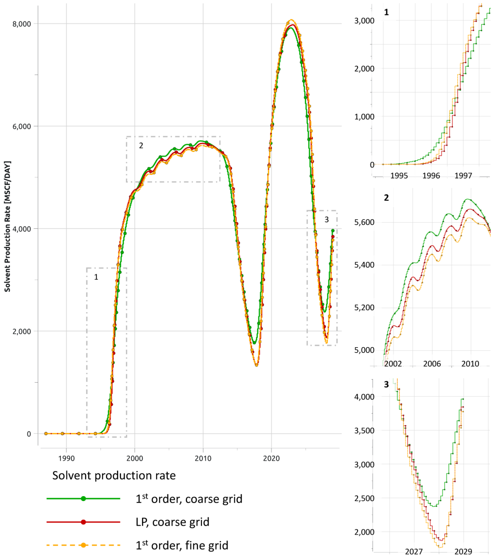

Cell Results:
SGAS
0.4417
0.3850
0.3300
0.2750
0.2200
0.1650
0.1100
0.0550
-0.0000
Cell Results:
SGAS
0.4382
0.3850
0.3300
0.2750
0.2200
0.1650
0.1100
0.0550
-0.0000
PROD
INJ
N(y)
E(x)

Oil and Gas production rate: zoom-in





Solvent production rate



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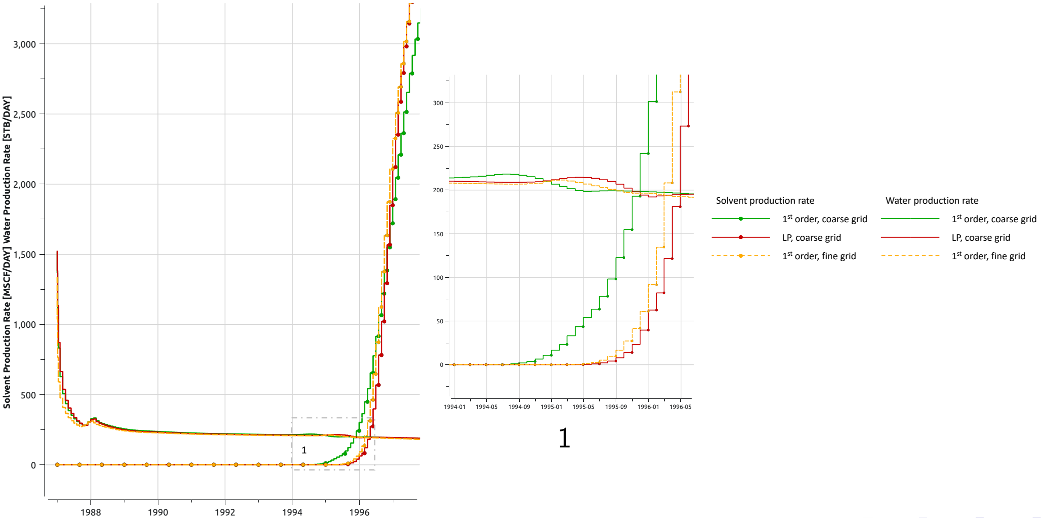
Production rates of water and solvent during the whole simulation on the right and zoom in on the left



Conclusions

- Showed that second-order method improves accuracy in front positioning and reduces smearing.
- Complexity of the reservoir can overshadow the effects gained by using a higher-order computational method.
- Verified the results with the first-order method on the refined grid, both for the medium-sized reservoir and the Norne test case.

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