Higher-Order methods in OPM. Based on the paper "A Second-Order Finite Volume Method for Field-Scale Reservoir Simulation"

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Introduction

The first-order finite volume (FV) is the default option in many standard reservoir simulators, both commercial and open-source.

+ It is robust.
+ Easy implementation.
  - Suffers from numerical diffusion.
  - Incorrect computations of the front position, components concentrations, water breakthrough, etc.
Introduction

To reduce the numerical diffusion and increase the accuracy, there are mainly two options:

- to refine the grid,
- to increase the order of the numerical method.
1. Second-order method with linear programming reconstruction
2. Explore the method’s capabilities in a realistic setting.
   - accuracy
   - verification in the absence of "true" solution
3. Run WAG and CO2 injection scenarios on the realistic test cases:
   - a medium-sized realistic reservoir with an unstructured corner point grid
   - an openly available Norne field mode
The models and the build instructions are available in the repository https://github.com/kvashchuka/second-order-opm-tests.

To run a test case with the second-order method, you need to enable certain flags:

```
./path_to_the_build_folder_of_opm-simulators*/bin/flow CASE_NAME --enable-higher-order=1 --enable-local-reconstruction=1 --reconstruction-scheme-id=3 --only-reconstruction-for-solvent-or-polymer=false
```
First- vs Second-Order FV Method

First-order FV method

\[
\lambda_{ij}^w = \begin{cases} 
\lambda_{ij}^-, & \text{if } (\nabla p_{w}^{n+1} - \rho_w g) \cdot n \geq 0, \\
\lambda_{ij}^+, & \text{otherwise}, 
\end{cases}
\]

Second-order FV method

\[
\lambda_{ij}^w = \begin{cases} 
L_{ij}^-, & \text{if } (\nabla p_{w}^{n+1} - \rho_w g) \cdot n \geq 0, \\
L_{ij}^+, & \text{otherwise}, 
\end{cases}
\]

where the linear reconstruction function has to satisfy the following requirements:

\[
L_{E_i}(x) := \lambda_{E_i} + \nabla L_{E_i} \cdot (x - w_{E_i}),
\]
\[
L_{E_i}(w_{E_j}) = \lambda_{E_j}, \quad \forall (E_i, E_j) \in \partial E_i.
\]
Second-order method with Linear Programming reconstruction

We want to minimize the total gaps between the reconstructed values and the cell-averaged values at all neighboring cells:

\[
\delta(L) := \sum_{(E_i, E_j) \in \partial E_i} |\lambda_{E_j} - L_{E_i}(w_{E_j})|.
\]  

(1)

The constraints are the following monotonicity conditions:

\[
\min\{\lambda_{E_i}, \lambda_{E_j}\} \leq L_{E_i}(w_{E_j}) \leq \max\{\lambda_{E_i}, \lambda_{E_j}\}, \forall (E_i, E_j) \in \partial E_i.
\]  

(2)
Second-order method with Linear Programming reconstruction

We solve the following LP problem:

$$\max \sum_{E_i, E_j \in \partial E_i} \text{sgn}(v_{E_j})(w_{E_j} - w_{E_i}) \cdot \nabla L_{E_i}$$

subject to $v_{E_j}^- \leq (w_{E_j} - w_{E_i}) \cdot \nabla L_{E_i} \leq v_{E_j}^+$,

where

$$v_{E_j}^- = \min\{0, \lambda_{E_i} - \lambda_{E_j}\},$$

$$v_{E_j}^+ = \max\{0, \lambda_{E_i} - \lambda_{E_j}\}.$$

The unknown vector $x$ is the gradient of the linear reconstruction $x = [\nabla L_{E_i}^x, \nabla L_{E_i}^y, \nabla L_{E_i}^z]^T$.

We use an all-inequality simplex method to solve the LP.
Norne: homogeneous (top) and heterogeneous (bottom)
Norne: homogeneous and heterogeneous

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volume of footallqualitiesdprudencec
dintehrereser, homogeneous Norne mode
1st order, homogeneous Norne model
1st order, heterogeneous Norne model
2nd order LP, homogeneous Norne model
2nd order LP, heterogeneous Norne model
Norne: verification with refined model
CO$_2$ injection on Norne

Figure: Positions of the wells in the Norne CO2 injection scenarios: left for the wells in the same compartment, middle - wells are separated by a fault, right - injection well in the corner.
Figure: Solvent production rate for the three scenarios of solvent injection on Norne.
**Figure**: The subplots zoom into the times of solvent arrival for each scenario: (1) Wells in the same compartment; (2) Wells separated by a fault; (3) Injection well in the corner.
A medium-sized realistic reservoir with an unstructured corner point grid
Oil and Gas production rate
Gas wave
Oil and Gas production rate: zoom-in
Solvent production rate
Production rates of water and solvent during the whole simulation on the right and zoom in on the left.
Conclusions

- Showed that second-order method improves accuracy in front positioning and reduces smearing.
- Complexity of the reservoir can overshadow the effects gained by using a higher-order computational method.
- Verified the results with the first-order method on the refined grid, both for the medium-sized reservoir and the Norne test case.
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